Hydraulic Engineering
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Lecture – 37
Non-Uniform Flow and Hydraulic Jump (Contd.)

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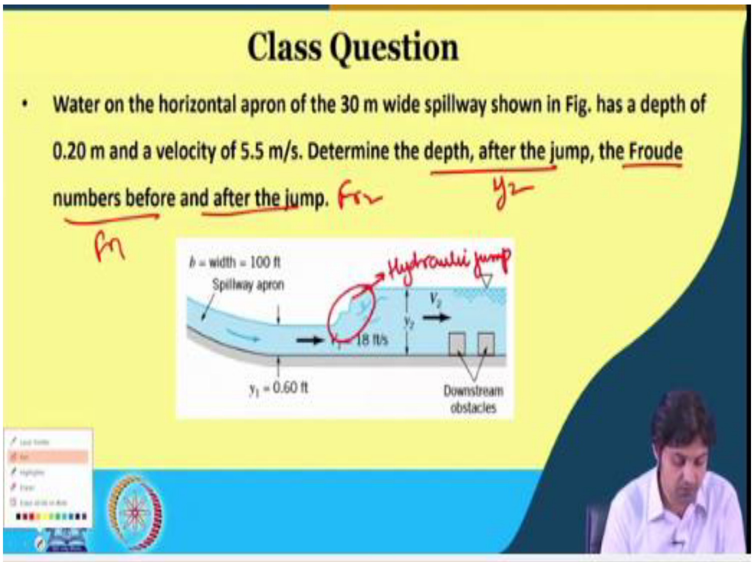

Welcome back students. So, we start with solving some questions that are related to hydraulic jumps, so rapidly varied flow. So, this entire lecture will be dedicated to solving some of the basics and a little more complex problems, those type of problems which you encountered in exams like GATE or IES.

So, we start with one problem as below, it says that water on the horizontal apron of 30 metre wide spillway. So, it is 100 feet or, sorry, that is 30 meter has a depth of 0 point, so 0.06 feet is equivalent to 0.20 metre and a velocity of 18 feet per second that means, 5.5 metres per second. The question is determining the depth after the jump, the Froude numbers before and after the jump. So, this is simple application of the formula that we have.

So, if you see, in this figure, so the water is coming and it is undergoing hydraulic jump, and we have to calculate y2, the depth after the jump, so that is y2, Fr1 and after the jump Fr2, simple.

So, we know the conditions here, at number 1, so it is pretty simple to calculate Froude number 1. We will go step by step, the way you should be solving to.

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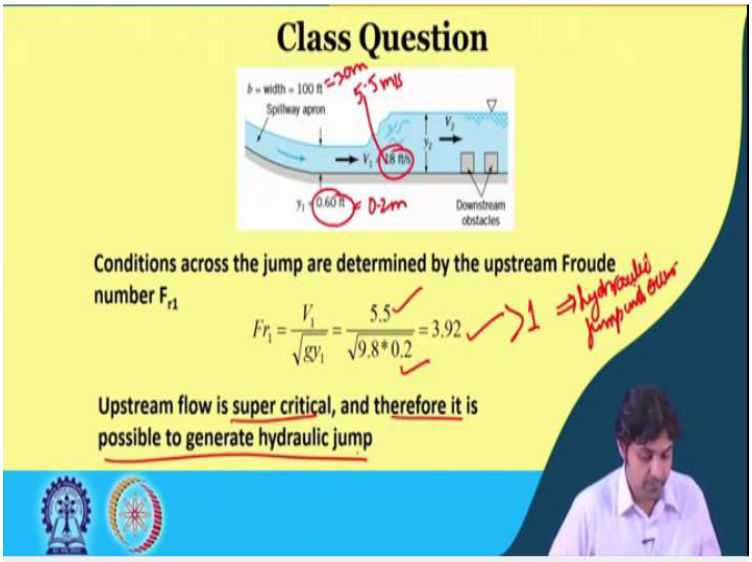

So, in this question, the conditions across the jump are determined by the upstream Froude number Fr1, that also we have to find out actually. So, Fr1 is given by V1 divided by under root gy1, V1 was given 5.5, y1 was given 0.2, you see, 18 feet per second means, 5.5 metres per second and this is 30 meter. So, the Froude number 1 comes out to be 3.92 and this is greater than 1, which means hydraulic jump will occur.

As I said that the upstream flow is supercritical and therefore, it is possible to generate hydraulic jump, first step.

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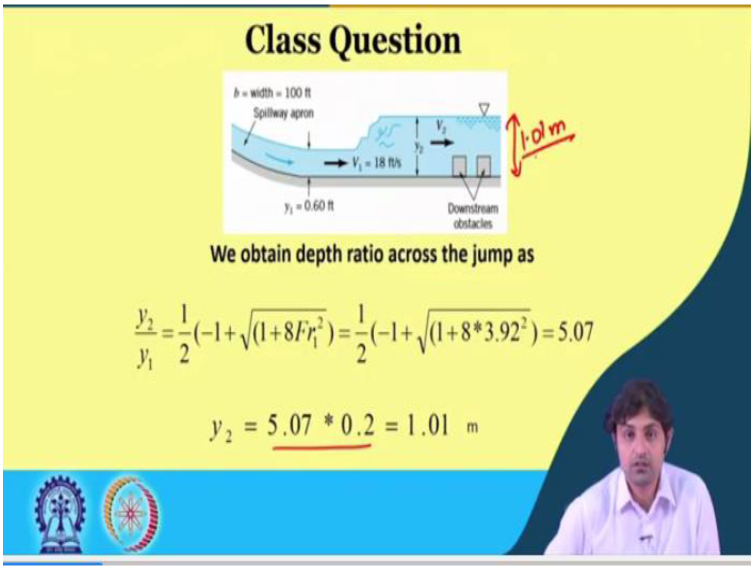

So, in the second step, we obtain depth ratio, we had the formula which said y2 by y1 is equal to 1/2 into multiplied by – 1 + 1 + 8 Fr1 square. Fr1 whole square which already we found out in the previous slide was 3.92, plug these values here, so what comes out is y2 by y1 is 5.07. Therefore, y2 is going to be 5.07 into 0.2 and that is 1.01 metre, so this is going to be 1.01 metre.

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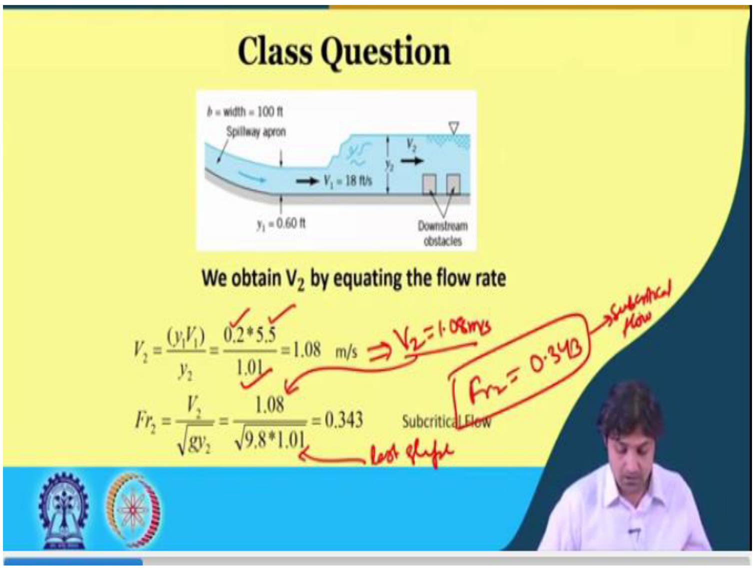

Now, the way we obtain V2 is by equating the flow rate. So, A1 V1 is equal to A2 V2, V gets cancel out, so V2 will be V1 y1 divided by y2, y1 was known from before, V1 was known from before, y2 we just calculated using that y2 by y1 equation of hydraulic jump, so V2 comes out to be 1.08 meters per second. So, therefore, Froude number at location 2 will be V2 by under root gy2. V2 we have calculated here, this was the reason we were calculating V2, for calculating the Froude number and this we calculated in the last slide.

So, Froude number 2 comes out to be 0.343, so this means subcritical flow. So, when the system goes hydraulic, when it undergoes hydraulic jump, the supercritical flow turns from supercritical turns into, the supercritical flow turns into a subcritical flow. So, Froude number is more than 1 and after undergoing the hydraulic jump, the Froude number becomes, I mean, Froude number becomes less than 1. So, the flow becomes subcritical so, as it is written here it is a subcritical flow.

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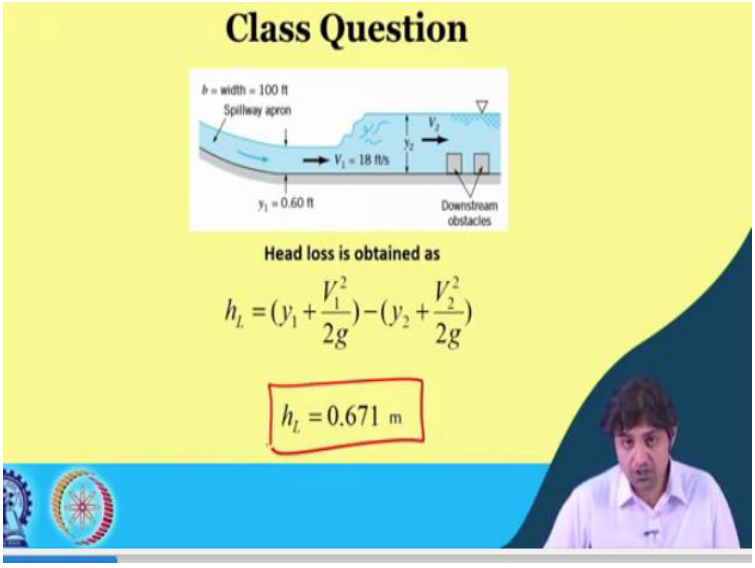

Now, we have to also obtain the head loss, that the energy loss. We simply use this equation, total energy at section 1 minus total energy minus section 2, y1 we have, we know from before, V1 we know from before, y2 we have calculated, V2 we have calculated. And after substituting in the values, you see, the head loss, that the energy loss, in terms of head is 0.671 metre.

This is the most simplest and the most common type of problems in hydraulic jump, which are the type of questions, you also will be expecting in your assignments and exams and competitive exams especially.

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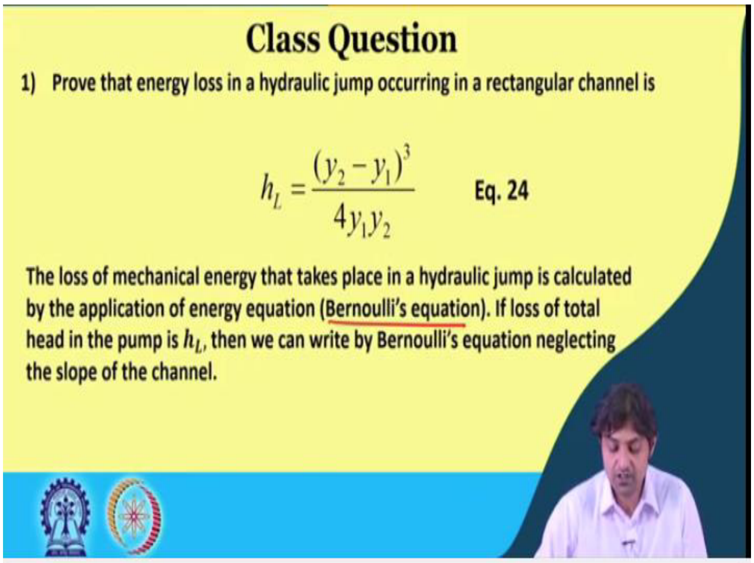

Now, we go to another question, say that prove that the energy loss in a hydraulic jump occurring in a rectangular channel is, so we would try to obtain head loss directly, in terms of y2 and y1. If we know the 2 depths, because sometimes in exams, in the objective type, you know, exams like IES and GATE, they simply give you y2 and y1 and asked you to calculate head loss. So, basically what you must do is, please learn, remember this equation.

In this particular question, we are trying to derive this, but you must remember this. We will derive this but in objective type of exam, it is very difficult to derive, I mean, all the time. So, basically, remember this equation. So, the loss of the mechanical energy that takes place in a hydraulic jump is calculated by the application of energy equation, Bernoulli's equation. If loss of total head in the pump is hl, then we can write by Bernoulli's equation neglecting the slope of the channel

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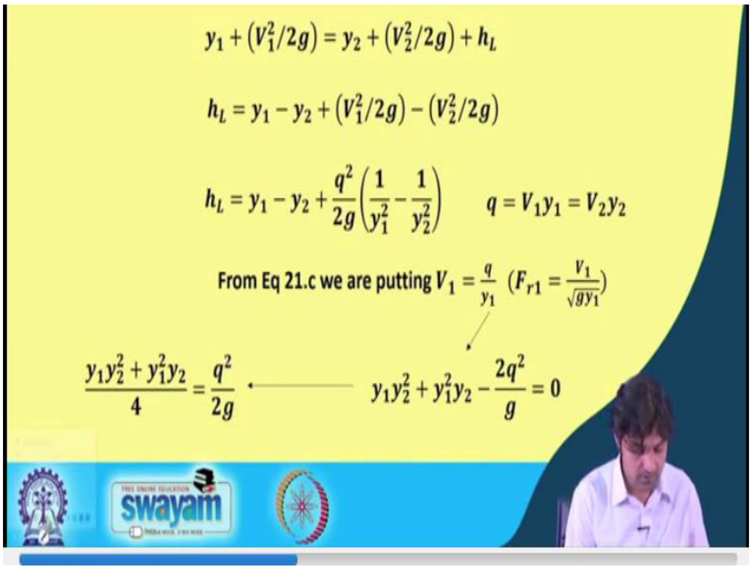

As y1 + V1 square by 2g is equal to y2 + V2 square by 2g + head loss, this is what we generally write. So, head loss can be written as, y1 – y2 + V1 square by 2g – V2 square by 2g. So, this remains same, so instead of V1 and V2, we write in terms of common quantity.

So, q can be written as V1 by y1 is equal to V2 by y2. So, we write, so V1, so we take out, sorry, very sorry, this equation, erase all, so this is the same, so V1 is q by y1. Okay, I just wrote the opposite, q by y2. So, this we substitute here and this we substitute here and therefore, this q square we take out common and this is 1 by y1 square minus 1 by y2 square.

Because q is equal to V1 y1 is equal to V2 y2. So, from equation number 21c, or we do not need to remember, V1 is actually q by, so you remember equation, we are putting V1 is equal to q by y1, Froude number 1 is V1 by under root gy1.

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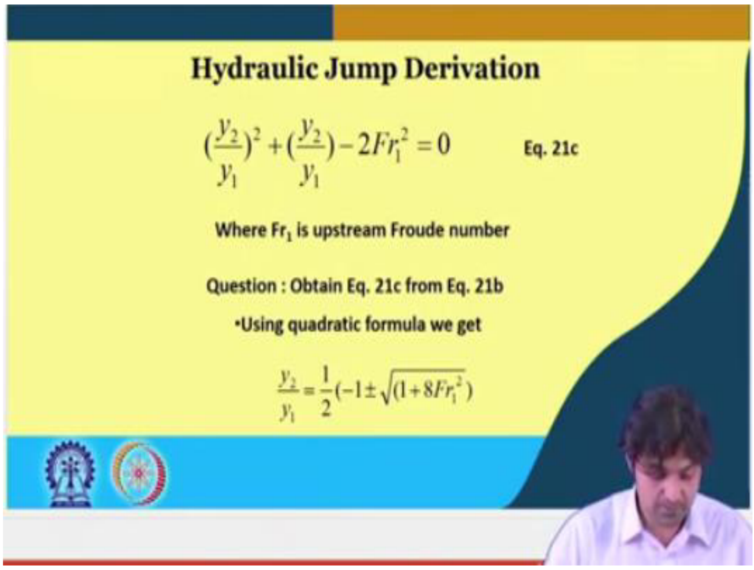

So, this one you see, you go back to the equation 21c, it is important to show you, so this one, this is the equation 21c. So, y2 by y1 + whole square + y2 by y1 - 2 Froude number 1 whole square is equal to 0. So, we can use this y2 by y 1 here, so we can simply write, y1 y2 whole square + y1 square y2 by 4 is equal to q square by 2g.

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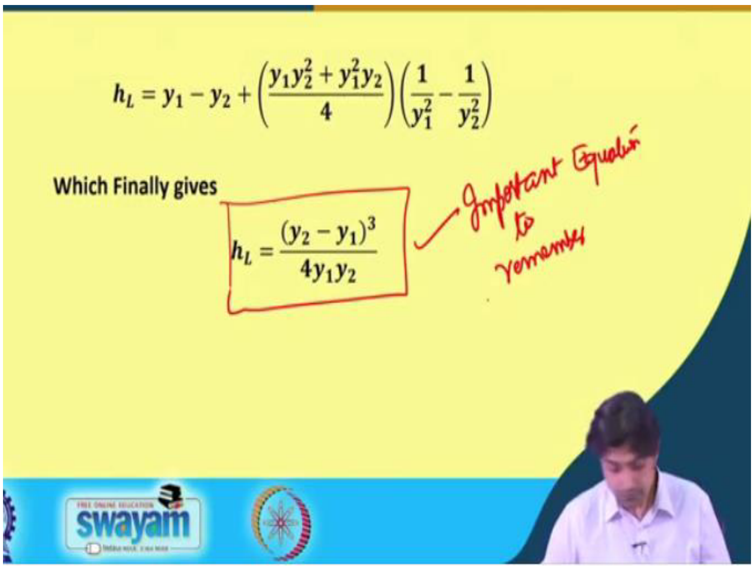

So, this equation becomes, so this q square by 2g is equal to this and we put it here. So, we get hl is equal to y1 – y2 + y1 into y2 square + y1 square y2 by 4 and multiplied by, which will finally give, if you solve this, this is going to give you hl is equal to y2 – y1 whole cube divided by 4y1y2, an important equation. I think you can, there are other ways of doing that as well but I think you should try this one at home, solving this one.

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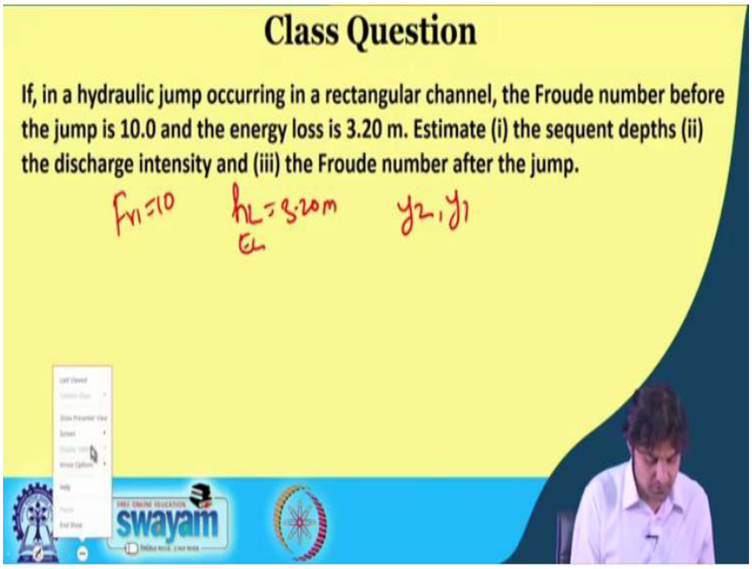

So, we go to another basic question, which says if in a hydraulic jump occurring in a rectangular channel, the Froude number before the jump is 10, so Fr1 is 10, and energy loss is 3.20 metre. What is the sequent depths? So, we are talking about y2 and y1 and then it asked about the discharge intensity and the Froude number, hl or El, whatever you want to call it. So, to solve this, we will take the help of white screen.

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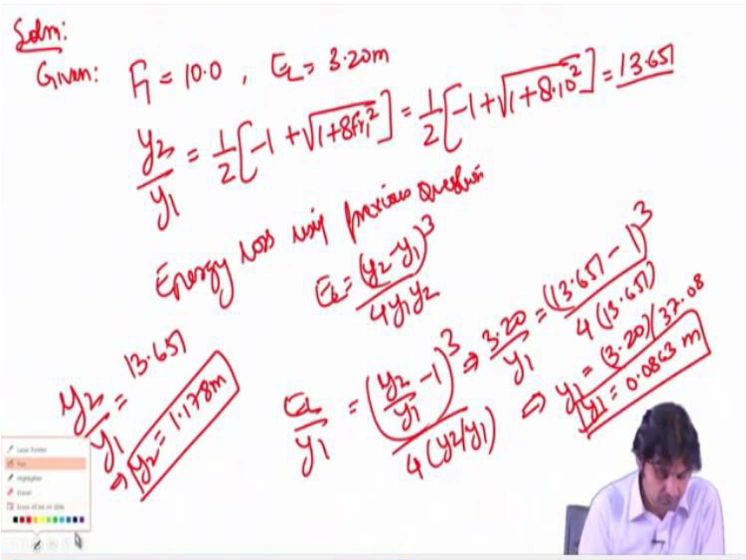

So, the things that are given to us is, Froude number or F1, let us say is 10.0 and energy loss or head loss is 3.20 meter. So, from the formula, you remember, we had found y2 by y1 is equal to 1/2 of – 1 + under root 1 + 8Fr1 whole square, so if we substitute – 1 + under root 1 + 8 is into

10 square and it will give us 13.651 and energy loss using previous question that I asked you to remember, EL is written as y2 – y1 whole cube divided by 4y1 y2.

So, if we say EL by y1 is y2 by y1 - 1 whole cube divided by 4 y2 by y1. So, EL we know is 3.20 divided by y1 will be 13.651-1 whole cube divided by 4 into 13.651 and on solving this, y1 is going to be 3.20 divided by 37.08, implies y1 is going to be 0.0863 meter, so 0.0863 meter. Now, y2 by y1 we already know, we have found out y1, so y2 by y1 came out to be 13.651, implies y2 is going to be 1.178 meter. So, the water level is increasing.

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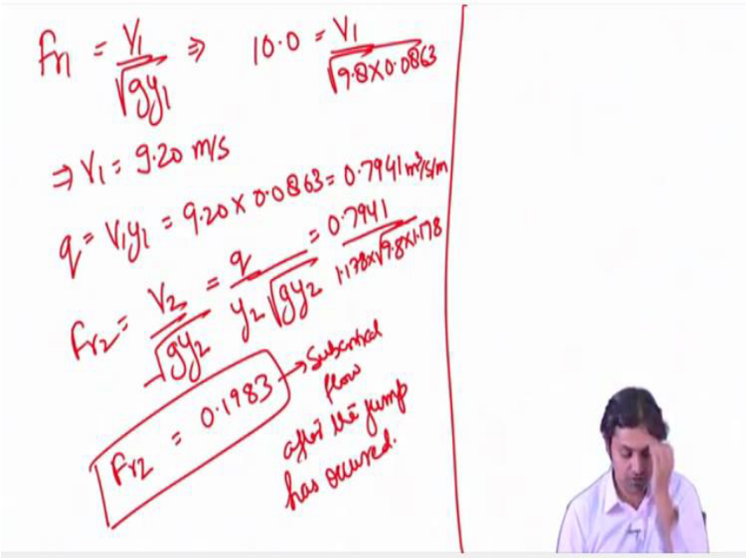

So, we have found out y2 and y1, so we also need to find out Froude number 1, that is, V1 by under root gy1. So, Froude number 1 we already know, 10 is equal to V1 under root 9.8 into 0.0863, implies V1 is going to be 9.20 meters per second, small q is going to be V1 y1, so 9.20 into 0.0863 is 0.7941 meter cube per second per meter and Fr2 is pretty simple, V2 by under root gy2. And V2 can be written, in terms of q by y2 under root gy2.

So, q we have already found out, 0.7941 and y2 is, y2 we found out was 1.178 under root 9.8 into 1.178. So, Froude number 2 is 0.1983, subcritical flow after the jump has occurred. So, this is a very simple problem, we using the formulas, so you see, how this formula of energy loss is equal to y2 -y1 whole cube divided by 4 y1 y2 is important.

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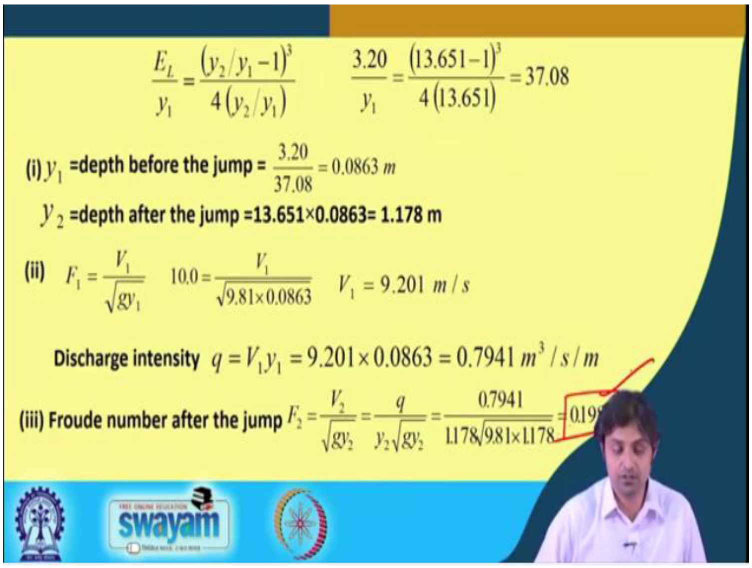

So, there is a solution as always here, you know, I have gotten it typed down. So, sequent depth ratio, so energy loss using this one, so after the jump Froude number is 9.2, discharge intensity, Froude number after the jump, same as we did it in the white board.

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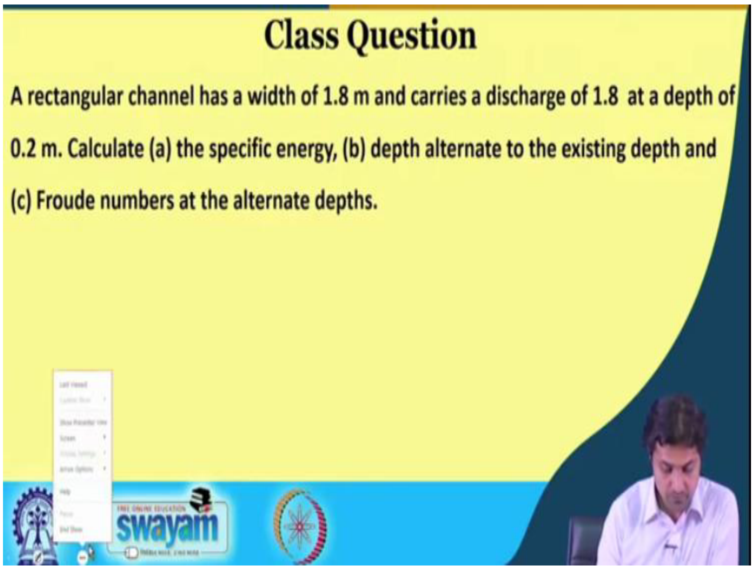

So, another question, it says a rectangular channel has a width of 1.8 meter and carries a discharge of 1.8 at the depth of 0.2 meters. Calculate specific energy, depth alternate to the existing depth and Froude number at alternate depths. So, in the beginning, we do not know what type of question this is. This is a hydraulic jump or not hydraulic jump but this is a very, this is a standard problem of open channel flow. So, we should start by, you know, writing down what are the things that we know from before, you know.

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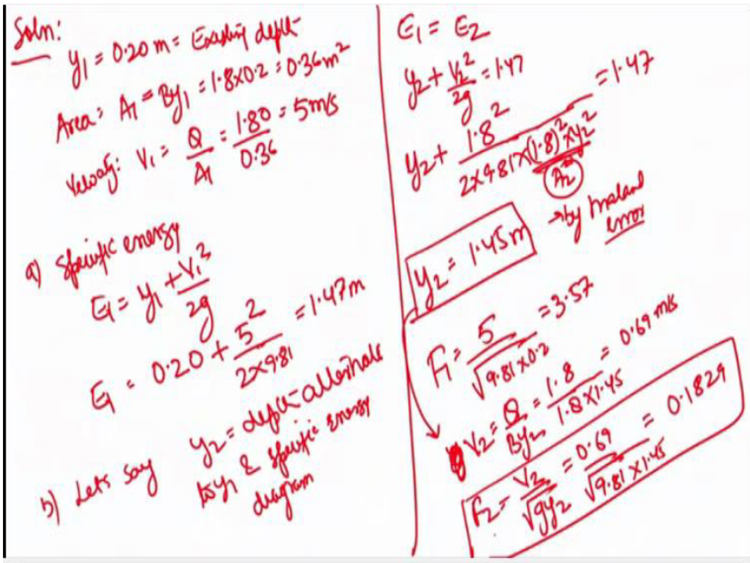

So to solve this, so what we know, we write it down, V1 is equal to 0.20 meter and that is the existing depth, area is A1 is By1, B is already given, width of 81.8 meter. So, it is 1.8 into 0.2, that is, 0.36 meter square. So, velocity V1 is simply, Q by A1, Q is given 1.80 and area we know is calculated 0.36 and this gives us to be 5 meters per second.

So, the first part is specific energy; E1 is equal to y1 + V1 square by 2g, so y1 we already know, plus V, V we have already calculated, 2 into 9.81, so this comes out to be 1.47 meter, so specific energy at 1. So, we have to calculate y2, so let say y2 is depth alternate to y1 and specific energy diagram. I would like to remind you, when we are dealing with specific energy, we found out a cubic equation, for a particular Q, there were existing 3 values of y. One was negative, which is generally neglected and the remaining one, y1 and y2 are called alternate depths.

In this question, one is already given to us and we are asked about the other one. So, the best way of solving this is through specific energy. So, we have calculated specific energy at one point and now, we are going to equate the specific energy at the other point. So, we write E1 is equal to E2, and E1 is, so E2 is y2, that we do not know, plus V2 square by 2g is equal to 1.4742 because that is E1 which we have already calculated, 1.47 actually, you will just write it down, it is better not going into too much.

So, y2 and V2 is what? V2 is similar to, you know, Q by A2. So, Q is 1 point, V2 square is Q AQ square divided by A2 square. So, we still write, 2.981 into, that is, 2g and then it is 1.8 because all the depth is, I mean, B1 into y1.

So, this is A square, A2 square, is equal to 1.47. So, if you solve for this, y2 will come out to be 1.45 meter, you can use by trial and error or you can, you know, it is not that big a problem. So, y2 comes out to be 1.45 meter. So, now, we know y1, we knew y1, now we have y2. So, Froude number at first location is going to be V under root 9.81 into 0.2, that is, 3.57 and for this y2, we need to find out first velocity.

So, V2 is going out to be, Q by By2. So, Q is 1.8, B is 1.8 and y2 is 1.45, so this comes out to be 0.69 meters per second. Therefore, F2 is V2 by under root gy2 and this comes out to be 0.69 into 9.81 into 1.45, this equals 0.1829. So, F2 comes out to be 0.1829. So, we have found using the concept of specific energy, everything that was required. So, F1, F2, you know, so we will close this.

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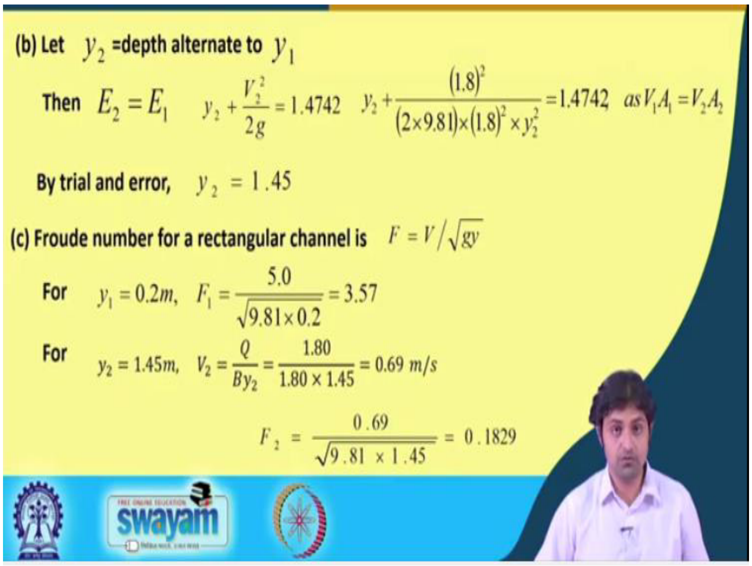

Here now, and go to the solutions that which we have solved actually. So, the velocity using the specific energy concepts, the same solution, Froude number. So, this will be of help to you when you revise the course.

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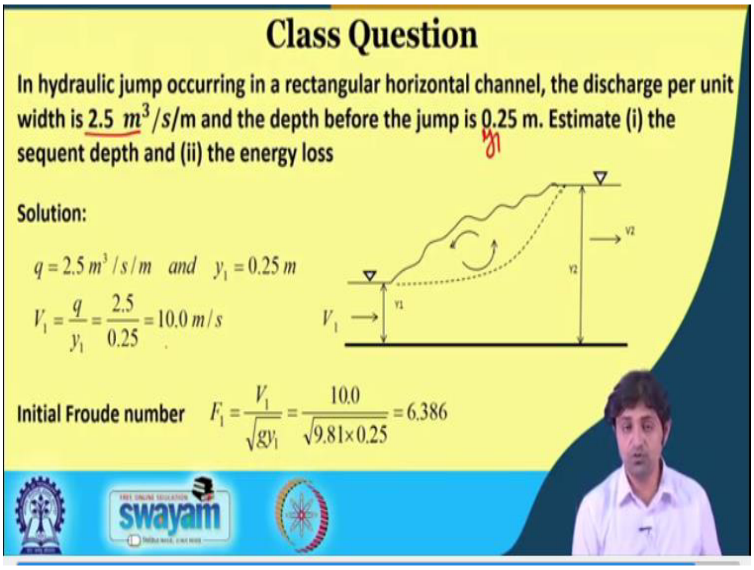

So, now, we are going to solve one last problem of this module and after that we will end this, you know, lecture. So, instead of going to the white board, I will take you through the slides itself and so. So, the question is, in hydraulic jump occurring in a rectangular horizontal channel, the discharge per unit width is 2.5 meter cube per second per metre. So, we have been given, I will explain here only.

The discharge per unit width, small q we have been given and we say that y1, we already know. Now, it is asking, estimate the sequent depth and the energy loss. So, the way it is to be done is, you see. We know small q, depth we already know, so this is the phenomenon, y1, V1, V2, y2. So, q is 2.5 meters cube per meters per second, y1 is 0.25 meters, already given. So, small v1, I mean, V1 is q by y1, which comes to be 10 metres per second.

And if we know V1, we can calculate the initial Froude number that comes to be 6.386. This means, we definitely will have a hydraulic jump because, I mean, we will have a hydraulic jump, we might, because Froude number is greater than 1. So, we know Froude number at location 1 now, we know y1, so we will be easily able to find out y2 by y1. Since, we know Froude number at location 1

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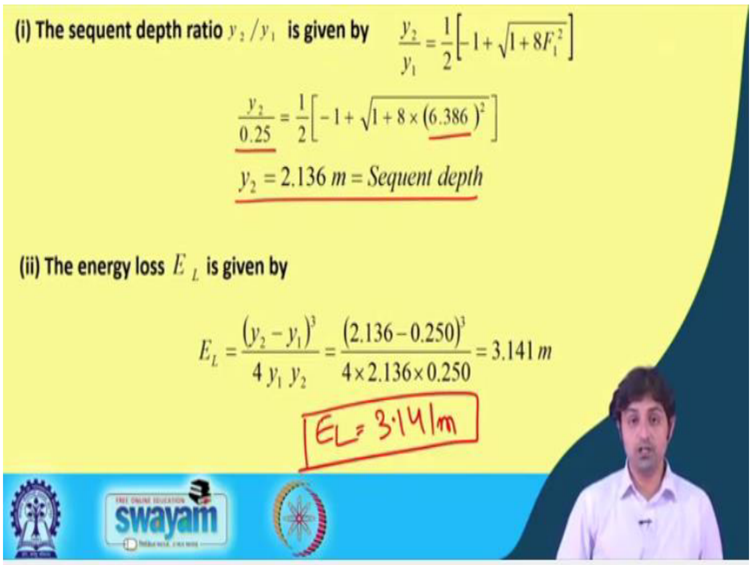

By the formula, y2 by y1 is equal to 1/2 -1 + under root 1 + 8 F1 square. This we have derived, you remember, and putting in the values, you see, Froude number came out to be 6.836, other things, y 1 we already, because we were given that it is 0.25 meter, so y2 will come out to be 2.136 meter, which is called the sequent depth. And now, the last part of this is we have to find out the energy loss EL.

And the energy loss EL is given by the formula; y2 – y1. So, we know y2, we know y1, so we can, we are easily able to find out energy loss, just in terms of y2 and y1. So, 2.136 - 0.250 whole cube divided by 4 y1 into y2 and the energy loss comes out to be 3.141 metre. So, this is the final question of this topic and with this we finished the module called as open channel flow that went on for 2 weeks.

So, in the next week we are going to deal with the pipe flow. We are going to go and see the viscous fluid flow, we will see concepts about computational fluid dynamics and we will close down with the wave mechanics, which is a typical example of in viscid fluids. So, thank you so much and I will see you next week. Bye.