Chapter 3

EQUILIBRIUM & REACTIONS

> To every action there is an equal and opposite reaction.
>
> Newton's third law of motion

3.1 Introduction

1 In the analysis of structures (hand calculations), it is often easier (but not always necessary) to start by determining the reactions.

2 Once the reactions are determined, internal forces are determined next; finally, deformations (deflections and rotations) are determined last[1].

3 Reactions are necessary to determine **foundation load**.

4 Depending on the type of structures, there can be different types of support conditions, Fig. 3.1.

Roller: provides a restraint in only one direction in a 2D structure, in 3D structures a roller may provide restraint in one or two directions. A roller will allow rotation.

Hinge: allows rotation but no displacements.

Fixed Support: will prevent rotation and displacements in all directions.

3.2 Equilibrium

5 Reactions are determined from the appropriate equations of static equilibrium.

6 Summation of forces and moments, **in a static system** must be equal to zero[2].

[1] This is the sequence of operations in the **flexibility** method which lends itself to hand calculation. In the **stiffness** method, we determine displacements firsts, then internal forces and reactions. This method is most suitable to computer implementation.

[2] In a dynamic system $\Sigma F = ma$ where m is the mass and a is the acceleration.

Structure Type	Equations					
Beam, no axial forces		ΣF_y				ΣM_z
2D Truss, Frame, Beam	ΣF_x	ΣF_y				ΣM_z
Grid			ΣF_z	ΣM_x	ΣM_y	
3D Truss, Frame	ΣF_x	ΣF_y	ΣF_z	ΣM_x	ΣM_y	ΣM_z
Alternate Set						
Beams, no axial Force	ΣM_z^A	ΣM_z^B				
2 D Truss, Frame, Beam	ΣF_x	ΣM_z^A	ΣM_z^B			
	ΣM_z^A	ΣM_z^B	ΣM_z^C			

Table 3.1: Equations of Equilibrium

3. The right hand side of the equation should be zero

If your reaction is negative, then it will be in a direction opposite from the one assumed.

16 Summation of all external forces (including reactions) is not necessarily zero (except at hinges and at points outside the structure).

17 Summation of external forces is equal and **opposite** to the internal ones. Thus the net force/moment is equal to zero.

18 The external forces give rise to the (non-zero) shear and moment diagram.

3.3 Equations of Conditions

19 If a structure has an **internal hinge** (which may connect two or more substructures), then this will provide an additional equation ($\Sigma M = 0$ at the hinge) which can be exploited to determine the reactions.

20 Those equations are often exploited in trusses (where each connection is a hinge) to determine reactions.

21 In an **inclined roller** support with S_x and S_y horizontal and vertical projection, then the reaction R would have, Fig. 3.2.

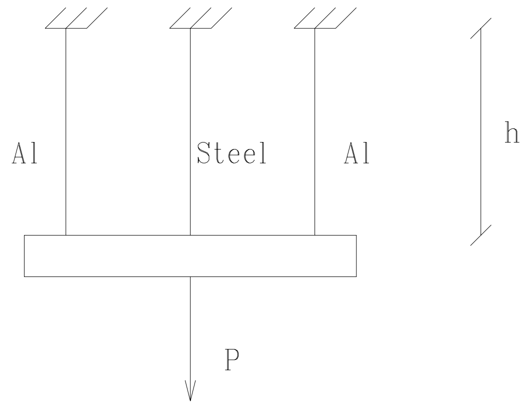
$$\boxed{\frac{R_x}{R_y} = \frac{S_y}{S_x}} \tag{3.3}$$

3.4 Static Determinacy

22 In statically determinate structures, reactions depend only on the geometry, boundary conditions and loads.

23 If the reactions can not be determined simply from the equations of static equilibrium (and equations of conditions if present), then the reactions of the structure are said to be **statically indeterminate**.

A rigid plate is supported by two aluminum cables and a steel one. Determine the force in each cable[5].



If the rigid plate supports a load P, determine the stress in each of the three cables. **Solution:**

1. We have three unknowns and only two independent equations of equilibrium. Hence the problem is statically indeterminate to the first degree.

$$\begin{aligned} \Sigma M_z = 0; \quad &\Rightarrow \quad P_{Al}^{\text{left}} = P_{Al}^{\text{right}} \\ \Sigma F_y = 0; \quad &\Rightarrow \quad 2P_{Al} + P_{St} = P \end{aligned}$$

   Thus we effectively have two unknowns and one equation.

2. We need to have a third equation to solve for the three unknowns. This will be derived from the **compatibility of the displacements** in all three cables, i.e. all three displacements must be equal:

$$\left.\begin{array}{rcl} \sigma & = & \frac{P}{A} \\ \varepsilon & = & \frac{\Delta L}{L} \\ \varepsilon & = & \frac{\sigma}{E} \end{array}\right\} \Rightarrow \Delta L = \frac{PL}{AE}$$

$$\underbrace{\frac{P_{Al}L}{E_{Al}A_{Al}}}_{\Delta_{Al}} = \underbrace{\frac{P_{St}L}{E_{St}A_{St}}}_{\Delta_{St}} \Rightarrow \frac{P_{Al}}{P_{St}} = \frac{(EA)_{Al}}{(EA)_{St}}$$

   or $-(EA)_{St}P_{Al} + (EA)_{Al}P_{St} = 0$

3. Solution of this system of two equations with two unknowns yield:

$$\begin{aligned} &\begin{bmatrix} 2 & 1 \\ -(EA)_{St} & (EA)_{Al} \end{bmatrix} \begin{Bmatrix} P_{Al} \\ P_{St} \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix} \\ \Rightarrow &\begin{Bmatrix} P_{Al} \\ P_{St} \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ -(EA)_{St} & (EA)_{Al} \end{bmatrix}^{-1} \begin{Bmatrix} P \\ 0 \end{Bmatrix} \\ = &\underbrace{\frac{1}{2(EA)_{Al} + (EA)_{St}}}_{\text{Determinant}} \begin{bmatrix} (EA)_{Al} & -1 \\ (EA)_{St} & 2 \end{bmatrix} \begin{Bmatrix} P \\ 0 \end{Bmatrix} \end{aligned}$$

■

[5]This example problem will be the only statically indeterminate problem analyzed in CVEN3525.