Hydraulic Engineering
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Lecture – 49
Viscous Fluid Flow (Contd.)

Welcome back students. So, this second lecture of viscous fluid flow, so where we are in the end going to derive the Navier-stokes equation but to be able to derive that we had to learn basic properties about the material derivative.

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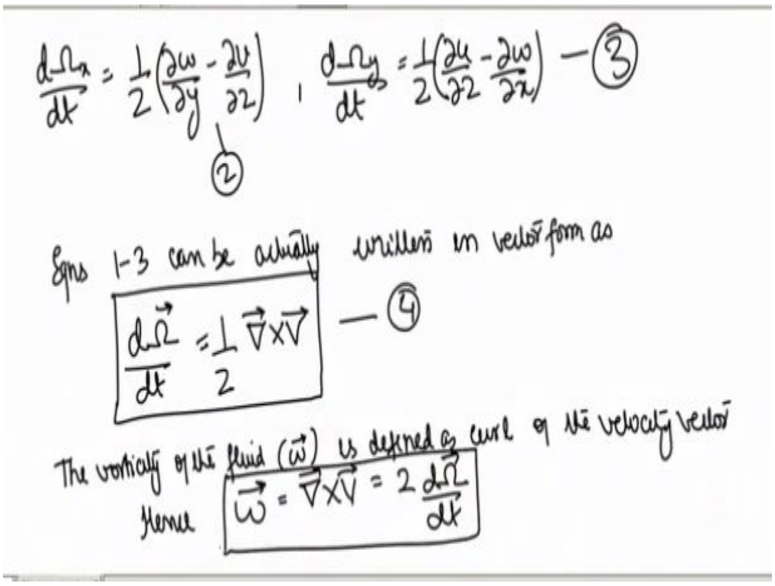

And also the rotation, what is rotation, how this angle is rotated, so we started from basics and until now we have derived that the rotation per unit time is given in a vector form as this, that was where we concluded in the last lecture. So, we will continue now, so this is, so there is a term called the vorticity. So, the vorticity of the fluid, this is actually the rate of rotation, this above equation, which let us say, I will name it as equation number 4.

So, the vorticity of fluid given by omega in vector form is defined as curl of the velocity vector. Hence, so this is the vorticity definition and this is; so, if you see, our angle of rotation here is half of the vorticity, it is actually $\frac{1}{2}\vec{\nabla}\times\vec{V}$ and vorticity is actually twice of that, so we can simply write 2, so this is another important result, which we have proved that vorticity is twice.

Because the rate of rotation we already derived and found out to be 1/2 of the curl, of the velocity vector, so we will go to the next page to continue.

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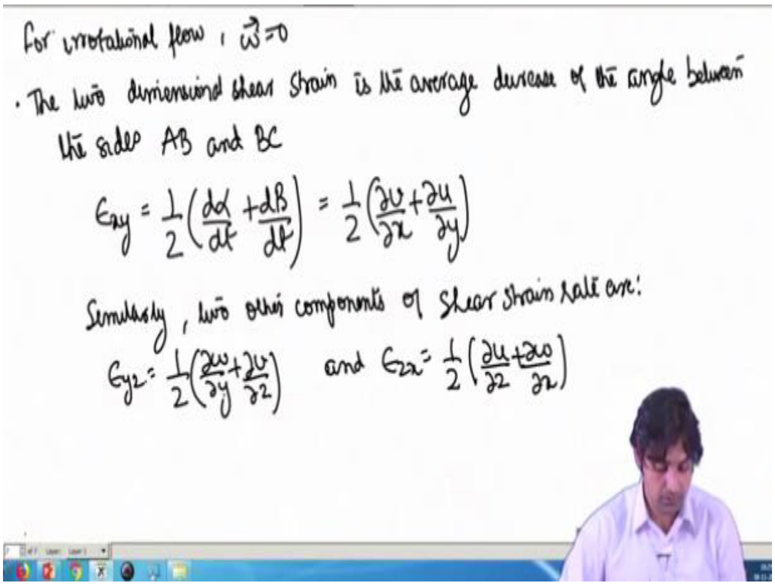

So, you know already for irrotational flow, omega is actually 0, because that is in; that is the vorticity vector should be 0. Now, the 2 dimensional shear strain is the average decrease of the angle between the sides AB and BC, in this figure.

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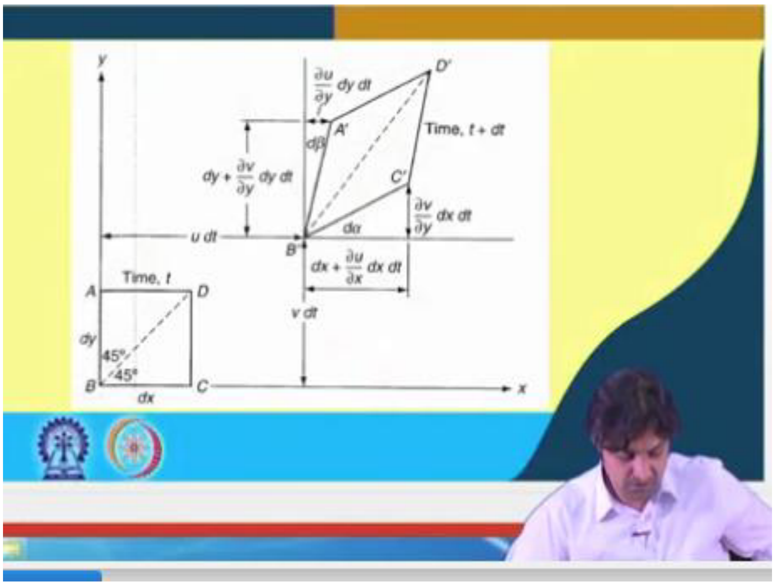

If you remember this figure, AB and BC, you know, so the 2 dimensional shear strain is the average decrease of the angle between the side AB and BC. So, we can define, this is the average, so that is the 2 dimensional shear strain is average of, so $\frac{1}{2}\left(\frac{d\alpha}{dt}+\frac{d\beta}{dt}\right)$, because that is the decrease of the angle or in other forms, we get $\frac{\partial v}{\partial x}$ because $\frac{d\alpha}{dt}$ was $\frac{\partial v}{\partial x}$ and this was $\frac{\partial u}{\partial y}$.

Similarly, 2 other components of, so this is actually shear strain rate, so using, drawing the analogy, ε_{xy}, we already wrote, so ε_{yz} can be written as, same way, $\frac{1}{2}\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}$ and ε_{zx} is equal to; so we have written ε_{yz} and ε_{zx} in a similar fashion.

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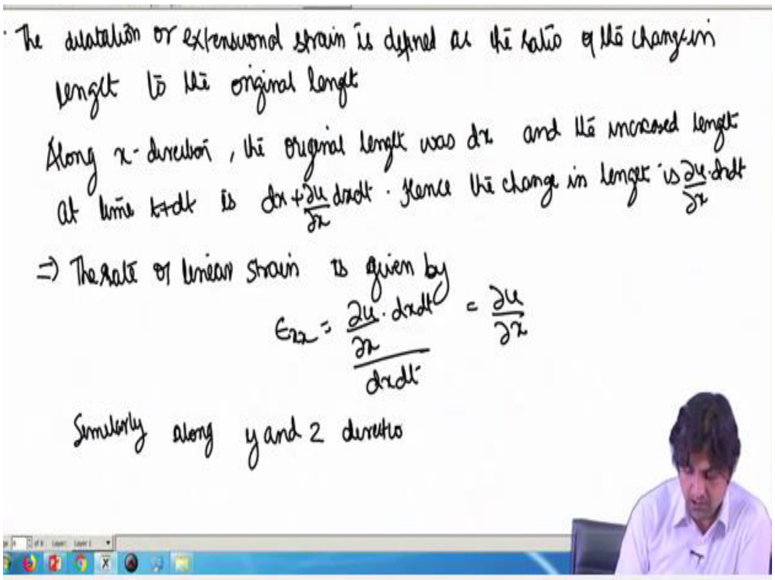

So, I will go to the next page, so the dilatational; dilatation or extensional, dilatation or extensional strain is defined as the ratio of the; see, the dilation or the extensional strain is defined as the rate of the change in length to the original length, this sort of a similar definition you would also remember from your thermodynamics class in class 12th. So, along x direction, the original length was dx.

And the increased length at time t + dt is which if you look at that diagram you would see del u del x into dx dt. Hence, the change in length is del u del x into dx dt. This implies, the rate of linear is given by, that was shear and this is linear, del u by del x into dx dt divided by dx dt, implied del u del x, so epsilon xx is del u del x.

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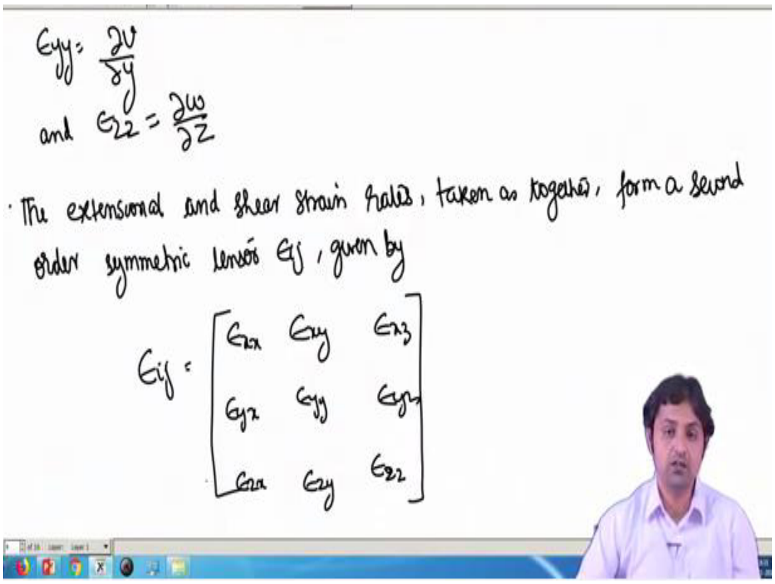

Similarly, along y and z direction, epsilon yy can be written as del v by del y and epsilon zz can be written as del w by del z. So, the extensional and shear strain rates, taken as together, form a second order symmetric tensor epsilon ij given by; so, epsilon ij, see, this is quite important, so we have actually found out the values in terms of del v, del v del x, del v del y, del v del z, another similar components.

So, what we have done here, we have written the total shear strain rates and we have found out that epsilon xx was del u del x, epsilon yy was del v del y, epsilon zz was del w del z and similarly, we also found out the shear epsilon xy, epsilon yz and epsilon zx, 1/2 of del v del x + del u del v del y, del u del y, so these are the relations which we have been successfully able to you know find.

So, in the next lecture, so I think this is the point is a good point to end the lecture today and in the next class, we will start by deriving some of the equations, starting with the equation of continuity and then going to momentum, proceeding to Navier-stokes equations. Thanks for attending the, listening to the lecture today and I will see you in the next class.