Hydraulic Engineering
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Lecture – 48
Viscous Fluid Flow

Welcome students. So, this is the week 10, lecture number 1, here we are going to study about the topic that is mentioned in this slide, this is about viscous fluid flow. Actually, you have, we have gone through this topic before but in a much more crude manner. The main objective of this module is going to be able to derive Navier Stokes equation from scratch, so how to start from the beginning and derive the Navier Stokes equation.

And we are expecting to dedicate around depends but at least 3 to 4 lectures in this module and our difference from the regular classes to this one is going to be that I will be teaching it by hand, we will take the help of slides as little as possible, because Navier Stokes equation is something that needs to be done by hand and in the derivation using the slide, it has been found out it is not helpful that much for this particular thing.

So, we will go very slow in this and we will complete this module in lecture 3 to 4, so now I will minimize this and use this board now.

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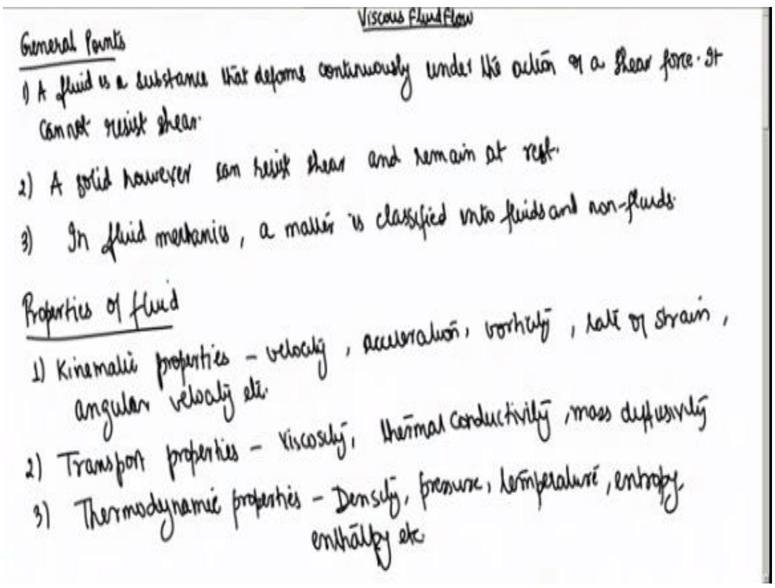

So, this is you know, the topic is viscous fluid flow so, I will be speaking too, because I think my writing might not be that much helpful to you, so to start with this one, some general, you know points; one is a fluid is a substance that deforms continuously under the action of a shear force, this means, it cannot resist shear. A solid however, can resist shear and remain at rest.

In fluid mechanics, a matter is classified into fluids and non-fluids. So, in thermodynamics, the normal definition is classification in solids, liquids and gases. But in fluid mechanics, it is fluids which consists of gases and liquids and non-fluids; non-fluids are mostly the solids. So, another basic revision is properties of fluid, so first is kinematic property, that is, velocity, acceleration, vorticity, rate of strain, angular velocity etc.

So, there are many other properties like this kinematic. Transport properties, you know, are viscosity, thermal conductivity, mass diffusivity. Thermodynamic properties like density, pressure, temperature, entropy, enthalpy etc. So, we will go to another page.

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4) Miscellaneous Properties – Surface tension, vapor pressure, eddy diffusion coefficient

Kinematic properties

a) Substantial derivative / Material derivative:

Let $Q(x,y,z,t)$ is any fluid property and $\vec{V} = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$ denotes the velocity field. Then total derivative of Q i.e dQ

$$dQ = \frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial y}dy + \frac{\partial Q}{\partial z}dz + \frac{\partial Q}{\partial t}dt$$

Hence, the time rate of change of the property Q is given by

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial Q}{\partial y}\frac{dy}{dt} + \frac{\partial Q}{\partial z}\frac{dz}{dt} + \frac{\partial Q}{\partial t}\frac{dt}{dt}$$

And the; there are some other miscellaneous properties. Miscellaneous properties includes surface tension, it is important to know these properties, at least know what those are vapour, pressure, see whatever properties I have just mentioned if you know what those are your knowledge of fluid mechanics is complete, eddy diffusion coefficient. So, apart from that so, these are the some of the properties.

So, now we go into little bit more detail in kinematic properties. So, the idea of this particular lecture today is that we are going to write what actually material derivatives are and we see the rotation you know how the fluid particle gets rotated and try to obtain the strain rates and those quantities that is the main objective of today's lecture. So, first thing is substantial derivative / material derivative.

So, we are just going to mention what that is, so if we say, let Q (x, y, z, t) is any fluid property and $\vec{V}$ is equal to $u(x,y,z,t)\hat{i}+v(x,y,z,t)\hat{j}+w(x,y,z,t)\hat{k}$ denote the velocity field. So, if we assume Q is any property, any fluid property and $\vec{V}$ given as $u\hat{i}+v\hat{j}+w\hat{k}$ denotes the velocity field, then total derivative of Q, that is, d Q, so dQ will be given as $\frac{\partial Q}{\partial x}dx+\frac{\partial Q}{\partial y}dy+\frac{\partial Q}{\partial z}dz+\frac{\partial Q}{\partial t}dt$.

Hence, time rate of change of the property Q is given by dQ/dt is equal to, so del Q del x is there and this dx/ dt, this becomes $\frac{\partial Q}{\partial x}\frac{dx}{dt}+\frac{\partial Q}{\partial y}\frac{dy}{dt}+\frac{\partial Q}{\partial z}\frac{dz}{dt}+\frac{\partial Q}{\partial t}\frac{dt}{dt}$. So, what we have done is we have divided this by dt.

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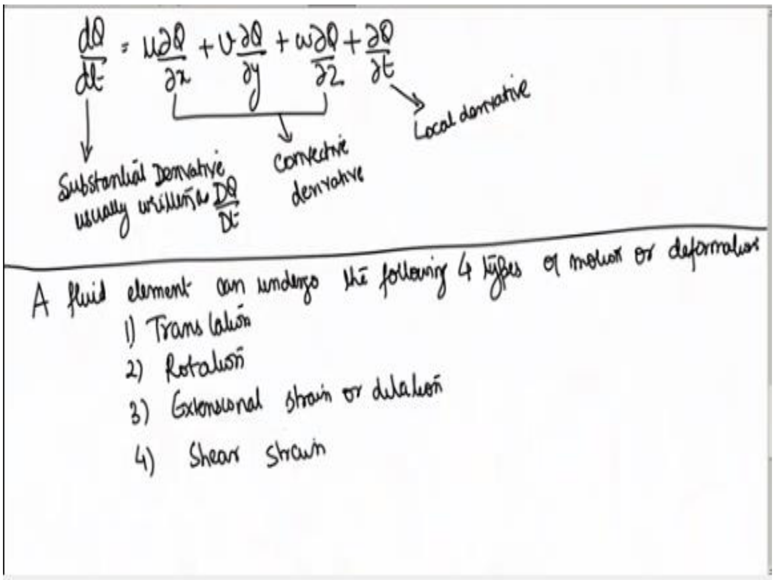

So, going to the next page, so finally we can write dQ/ dt is given by, because dx dt becomes u; $\frac{\partial Q}{\partial x}u+\frac{\partial Q}{\partial y}v+\frac{\partial Q}{\partial z}w+\frac{\partial Q}{\partial t}$. So, this dQ/ dt is called substantial derivative, usually written as dQ dt. These 3 are convective derivative and this is local derivative, so this is substantial or material derivative, this is convective derivative and this is the local derivative.

So, starting with a new, you know, so if there is a fluid element can undergo the following 4 types of motion or deformation. What are those? Of course, it can do translation; translation is moving from one point to the other, it can do rotation; it can do extensional strain or also dilation and shear strain.

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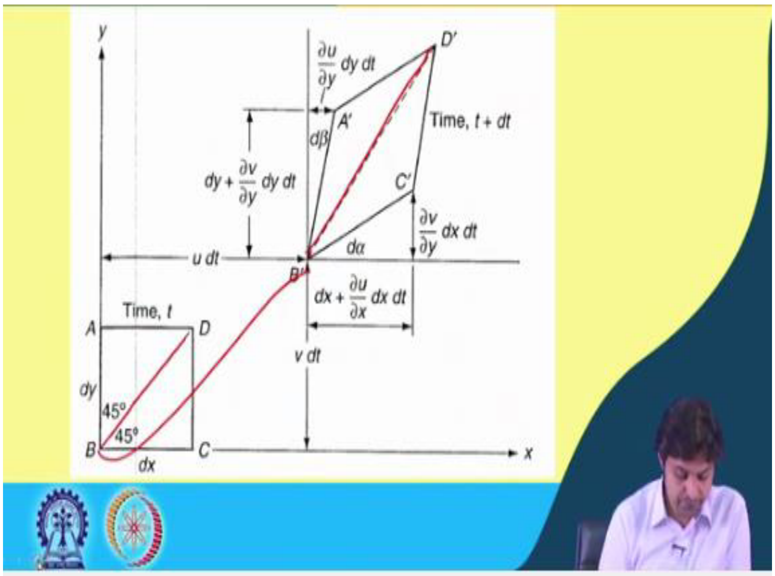

So, we are going to derive something which we, so we are going to derive the strain rates, so one thing I would like to take your attention to, is this figure. So, we consider a fluid element A, B, C, D, so we consider this element A, B, C, D and that moves in an xy plane, the position of this element at times t, this is position at time t and its position at time t + dt is shown like this.

So, now with reference to the above figure, we can observe the following transformations, I mean, of transformation or deformation in the fluid element within the time interval of t + dt. See, we can see that B has translated from B to B dash, we can also see that this diagonal BD has rotated anti-clockwise to B dash, D dash, you see that, and we can also see the dilation or the extensional strain of the element.

Because of rotation, you know, these elements, some of these will dilate and some of them will elongate. So, now using this figure we are going to write down, so we will keep this handy and start deriving.

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From this figure

$$\tan d\alpha = \frac{\frac{\partial v}{\partial x} dx \cdot dt}{dx + \frac{\partial u}{\partial x} dx \cdot dt} \quad \text{and} \quad \tan d\beta = \frac{\frac{\partial u}{\partial y} dy \cdot dt}{dy + \frac{\partial v}{\partial y} dy \cdot dt}$$

for infinitesimally small time interval dt

$$d\alpha = \lim_{dt \to 0} \left[\tan^{-1} \left[\frac{\frac{\partial v}{\partial x} dx\, dt}{dx + \frac{\partial u}{\partial x} dx\, dt} \right] \right]$$

$$\Rightarrow d\alpha = \lim_{dt \to 0} \left[\tan^{-1} \left(\frac{\frac{\partial v}{\partial x} dt}{1 + \frac{\partial u}{\partial x} dt} \right) \right] = \frac{\partial v}{\partial x} dt$$

So, we can see from the figure that I showed you, we can say tan d alpha is equal to del v del x into dx dt divided by dx + del u del x dx dt. So, let me go back to this figure and try to explain you some of these components here. You see, so this was length dx, when it goes there, you know, if we consider in time dt and with the velocity u.

So, this entire length will become dx plus, so change of velocity in x direction, if it is del u del x, if it is del u del x, then the total change, this length is going to be del u del x into dx into you know dt. So, this is the total distance that this is going to be there and the translation that this will have, this one is going to be only, see this was at origin, I mean, this was at 0 before.

So, it will be del v del y, so it would have also, it has traversed over a length x, so dx into dt, similarly from here to here you see dy, so it was already at y, so this length is dy plus the rate of change of v in y direction, so del v del y into dy dt. Similarly, this will be only del u del y into dy dt. I hope you have understood, so there are angles d alpha and d beta which we are going to calculate.

So, the translation because it is u dt, you see, this translation from this point to this point in time dt is considered to be u dt and this one it has a velocity u in this direction and this one, this fluid particle has a velocity v in this direction, so this has translated to, u dt and v dt, if it was 0, 0 here. So, this is done using geometry only, so a fluid particle having velocity u and v respectively in x and y direction translates that rotate also you know because of the shear.

So, this particular figure we are going to see, we are going to derive some important results about it. So, tan d alpha you see, I am not going to open it all the time, so I will show you here, so this is d alpha here, so this is d alpha and this is d beta. So, tan d alpha was del v del x into dx dt divided, just simple geometry. And what is going to be tan beta?

Tan d beta is going to be del u by del y into dy dt divided by dy + del v by del y into dy dt, simple trigonometric rule. So, for infinitesimally small time interval dt, if the time interval is very small that is dt, we can obtain tan d alpha is d alpha is limit, dt goes to 0 tan inverse of del v by del x into dx dt divided by dx + del u del x into dx dt.

This implies, d alpha is limit dt goes to 0 tan inverse, so we divide by dx, so this will be del v del x into dt divided by 1 + del u del x dt, we just cancelled dx from up and down and in this case, what is going to happen is; because dt is very small, this is going to be 1 plus, this is going to be 1 and it will be del v by del x because this will be approximately equal to 1, del v dt.

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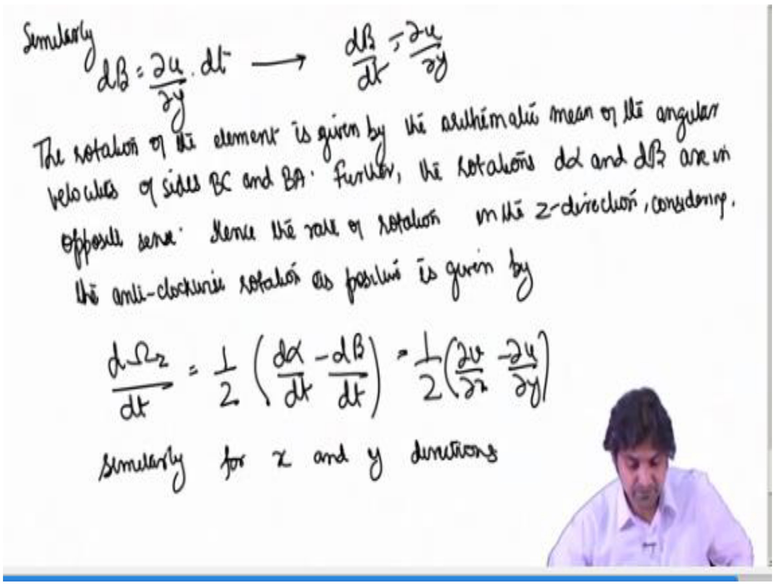

So, similarly, the way we obtained d alpha, d beta is also going to be del u del y into dt, this is another important results, del u and del beta. So, now, the rotation of the element is given by the arithmetic mean of the angular velocities of sides BC and BA. Further, the rotations d alpha and d beta are in opposite sense. Hence, the rate of rotation in the z direction considering the anti-clockwise rotation as positive is given by; so rate of rotation in the z direction is given by d in z direction, is given by, simple.

So, the rotation of the element it was given by the arithmetic mean of the angular velocities of sides BC and BA, which is 1/2 d alpha dt - d beta dt, here we have assumed that the anti-clockwise rotation is positive, so because d alpha by dt and d beta by dt, we have found out from the previous, you see, d beta, so if you see using this one for example, d beta by dt can be written as del u by del y.

So, we can simply write, del alpha by del t is del v by del x and this will be del u by del y.

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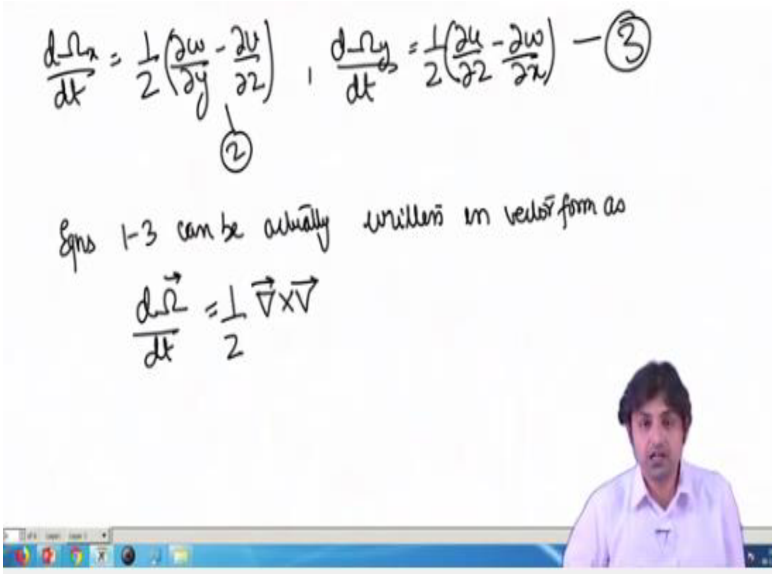

So, similarly for x and y directions, simply we write sigma x by dt is equal to 1/2, so we have found out for z, same way we can find out for del w by del x, sorry, del w by del y - del v by del z and so d sigma y by dt is 1/2 del u by del z - del w by del x. So, let us call this equation number 3 and the other 2 equations, you know, so let us say this is 1 and this was equation number 2.

So, equations 1 to 3 can be actually written in vector form as, because there are 3 components, we can simply write; is equal to 1/2, so I would like to close the lecture at this point and we will resume from this particular page in our next one. Thank you so much. See you in the next lecture.