

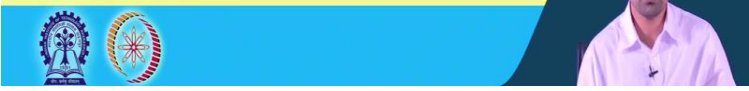
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Lecture # 62
Introduction to wave mechanics (Contd.)

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CELERITY IN DIFFERENT WATER DEPTH CONDITIONS:			
Classification	d/L	$2\pi d/L$	$\tanh(2\pi d/L)$
Deep Waters	$>1/2$	$>\pi$	~ 1
Intermediate waters	$1/20$ to $1/2$	$\pi/10$ to π	$\tanh(2\pi d/L)$
Shallow waters	$\leq 1/20$	0 to $\pi/10$	$\sim 2\pi d/L$

Classification of ocean waves according to water depth



Welcome back student to the fourth lecture of this final module about inviscid flow or we are going to derive we everywhere actually deriving the equations of velocity potential here. So, to proceed forward, we are going to see we have already derived the dispersion relationship. So, one thing you must remember that the complex derivation you do not need to remember the only thing that you need to remember are the end results.

For example, the velocity potential ϕ and the dispersion relationship. Now, whatever results we have obtained, we are going to apply in certain situations such as different water depth. So, this is a table, but first without going into too much you know detail first I would like to say there is something which we have said deep waters, we have said intermediate waters and we have shallow waters.

So, what are the definition of deep water deep water means that the depth of the water divided by the wavelength in which the you know the wavelength of the wave if it is greater than half. That means it is deep water if this ratio d by L is less than 1 by 20 or 0.05 . Less than 0.05 that the

depth of the water and the way the ratio of the depth of the water to the wavelength of the wave it is less than 1 by 20. That means we are in the shallow waters. And what the value in between 0.5 and 0.05 we call it intermediate waters, that is our definition that is how we have defined what a deep water is, and water shallow water and what are the intermediate waters are.

So, if in case of deep water, if d by L is greater than half, then we will have $2\pi d$ by L will be greater than π , what you do simply is 2π just the d by L is greater than half. So, $2\pi d$ by L will be greater than 2π into half. So, this for Deepwater 2 by 1 will be greater than π . Similarly, for intermediate waters this value is going to be $2\pi d$ by L will be between π by 10 and π and for below that in shallow waters.

It will be from 0 to π by 10 very simple to calculate yourself. So, for this condition when in $2\pi d$ by L is greater than π then $\tanh$ of $2\pi d$ by L can be approximated as 1 you can use you are trigonometric calculators to find that for shallow waters if $2\pi d$ by L is 0.2π by 10 we can approximate that is $2\pi d$ by L So, $\tanh$ will go away whereas, in case of intermediate it will remain the same $2\tanh 2\pi d$ by L .

So, these are 3 values of $\tanh 2\pi d$ by L that we get and this is the classification of ocean waves according to the water depth deep water Intermediate water in shallow waters.

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Deep water conditions:

- In case of deep waters Eq (2.30) becomes

$$c_0 = \sqrt{\frac{gL_0}{2\pi}} \quad \text{since, } \tanh kd = 1$$

and Eq (2.31) becomes

$$c_0 = \frac{gT}{2\pi} \quad (2.33)$$

Or $L_0 = \frac{gT^2}{2\pi} \quad (2.34)$



Let us say the deep water condition in case of deep water equation 2 point 30 so, what was equation number 2 point 30. So, equation 2 point 30 was $C^2 = g/k \tanh(kd)$ now we go back to this. So, c not because since $\tanh(kd) = 1$ c not will become under root gL not by 2π . And equation 2 point 31 was we see not becomes gT by 2π or a lot can also be written using this equation here, g by $2\pi T^2$ or we can also write L not is g by 2π is 9.8 divided by 2 into 3.14 into T^2 are lot can be written as $1.56 T^2$.

So, in deep water if we know the time period of the wavelength can be simply derived as 1.56 times T^2 a very important result also the wave speed c not gT by 2π 1.56 into it T . You see how we have applied the findings into case of deep water.

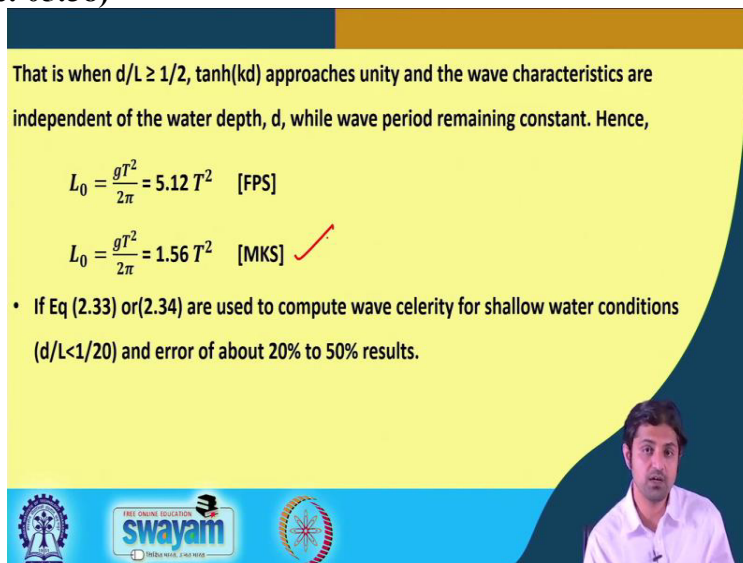
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That is when $d/L \geq 1/2$, $\tanh(kd)$ approaches unity and the wave characteristics are independent of the water depth, d , while wave period remaining constant. Hence,

$$L_0 = \frac{gT^2}{2\pi} = 5.12 T^2 \quad [\text{FPS}]$$

$$L_0 = \frac{gT^2}{2\pi} = 1.56 T^2 \quad [\text{MKS}]$$

- If Eq (2.33) or (2.34) are used to compute wave celerity for shallow water conditions ($d/L < 1/20$) and error of about 20% to 50% results.



That is when d by L is greater than half $\tanh(kd)$ approaches unity and the wave characteristic are independent of water depth. One important thing to note is that here the term d does not appear in this equation because the term that contained to d was $\tanh$ and this the $\tanh$ term goes to work in case of Deepwater. Therefore, all the way of characteristic are independent of the water depth, this is an important result.

So, a lot is in FPS so, this is a different unit within SI unit which is the one that we are going to use will be 1.56 times T^2 , this is the meter kilograms second unit SI unit. So, if equation 2.33 and 2.34 are used to compute wave celerity for shallow water condition. So, if we say that we are going to use the same equation for shallow waters, we are going to get some errors of the

order of 20 to 50%, which is not good because we need to get the correct value of the velocities in shallow water. Shallow water occurs near the coast for example.

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Shallow water conditions:

- When $kd = \frac{\pi}{10}$, $\frac{d}{L} \leq \frac{1}{20}$

$$C^2 = \frac{gL}{2\pi} \tanh(kd)$$

- Hence $\tanh(kd) \approx kd = 2\pi d/L$ $C^2 = \frac{gL}{2\pi} \cdot \frac{2\pi d}{L}$ $C^2 = gd$

$$C = \sqrt{gd} \quad (2.35)$$

This relation shows that when a wave travels in shallow waters wave celerity depends only on the water depth.

Now, what are the shallow water conditions $kd = \pi$ by 10., that is what we said $2\pi d$ by L that is what we found out from the table. Therefore, c square can be written as gL by $2\pi \tanh kd$ then if k and hkd can be written as $kd = 2\pi d$ by L . So, this will become C squared will become gL by 2π into 2 by d by L right. So, 2π 2π gets canceled and c squared = gd same as what we have written here So, C will be under root gd .

So, in shallow waters the waves speed will depend on nothing but only the water depth in deep water it was not dependent on the water depth but in shallow water, the only thing on which the wave speed depends on is the water depth if you recall this is so, same equation as we got in open channel flow module remember we got $C = \sqrt{gd}$ by applying the principles of continuity momentum and energy balance.

So, in shallow waters wave speed is under root gd where gd the depth of the water. So, as I said this relationship shows that when a wave travels in shallow water waves celerity depends only on the water.

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Relationship between d/L and d/L_0

- It can be shown by dividing the Eq.(2.31) by Eq.(2.33) and dividing Eq. (2.32) by Eq.(2.34) that $C/C_0 = L/L_0 = \tanh kd$

Multiplying both sides by d/L , then

$$d/L_0 = d/L \tanh kd \quad (2.36)$$

The relation between d/L and d/L_0 is given in the wave Tables



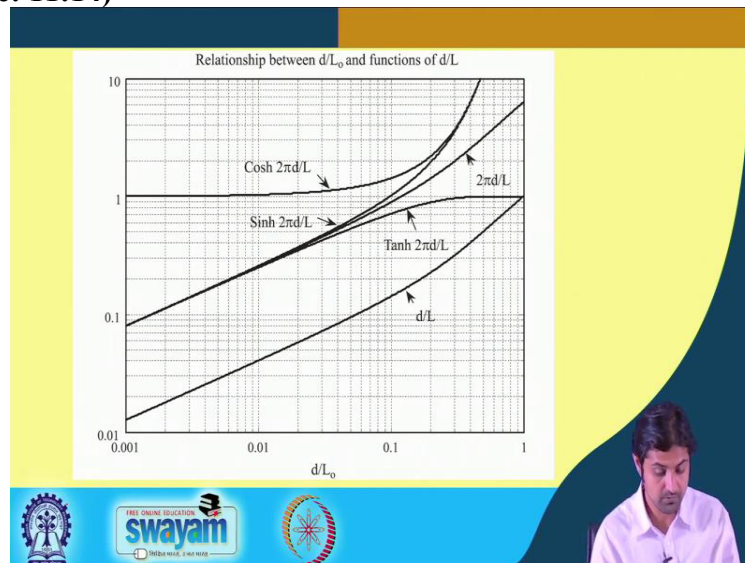

Now, relationship between d by L and D by L not if you divide the equation 2.31 by equation 2.33 and equation 2.32 by equation 2.34 we can simply get so, 2.31 and 2.32 are the equations which are general form 33 and 34 was doubt of deep water. So, if we divide those we can get C by C not = L by L not = $\tanh kd$. You can try this on your own. Now, if we must multiply both sides by d by L , we can get d by L lot = d by L tan hkd .

Now, the relationship between D by L not and d by L and D by a not is given in wave tables also. So, this is something if you want to calculate there is something called wave tables, which I would not show you in this course. But if you need to solve you can always use trial and error method for solution of the desperation relationship, but most of the question that you will be getting will be in your assignments will be based either on deep water or very simple that you can use the equations like there is present relationship or the velocity potential.

Where a lot of iterations and intermediate water determination would be required, I would not be giving those type of problems for you to solve, because the idea here is that you understand the basics of wave mechanics and therefore, if I give you more questions about intermediate waters, it will only complicated calculations. Therefore, the use of wave tables will not be this is not encouraged.

However, I can try to give you questions in your assignment 1 or 2 which will show exactly how to use the iterative formula, iterative procedure. And I can also post some couple of questions in the forum. Where you are going to use those iteration method for solving the wave amplitude or the intermediate water depth properties.

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So, this is the curve which is which denotes the relationship between d by L and D by L not. So, the right hand side then this shows d by L not.

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LOCAL FLUID PARTICLE VELOCITIES AND ACCELERATION UNDER PROGRESSIVE WAVES

- In the evaluation of wave forces on offshore structures it is desirable to know the fluid particle kinematics that is velocity and acceleration.

We know

$$\phi = \frac{ag}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \cos(kx - \sigma t) \quad (2.37)$$

- The horizontal water particle velocity or orbital velocity u is given by

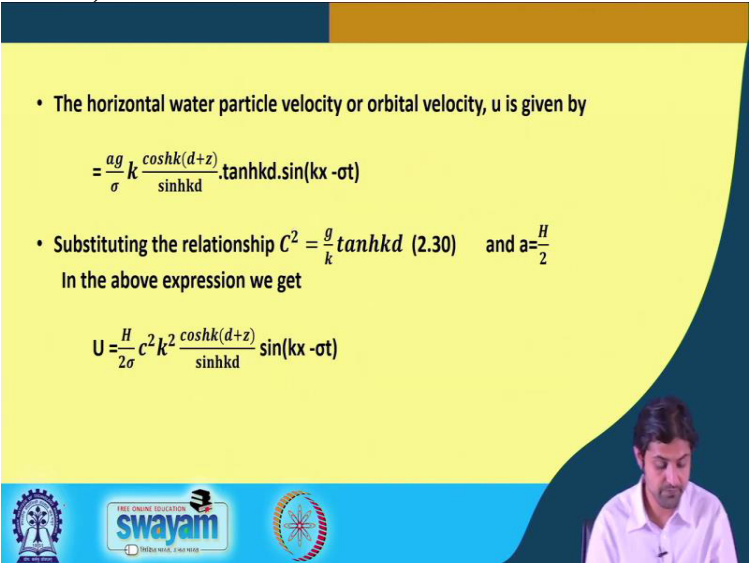
$$u = -\frac{\partial \phi}{\partial x} = \frac{ag}{\sigma} k \frac{\cosh k(d+z)}{\cosh kd} \sin(kx - \sigma t) \quad (2.38)$$

So, see proceeding forward now, we have found out the velocity potential we have found out the dispersion relationship now it is time to form find out the local fluid particle velocities and

acceleration under progressive waves these So, in the evaluation of I mean first more importantly, why do we need to find out these velocities and acceleration because in evaluation of waveform on offshore structures, it is desirable to know the fluid particle kinematics that is velocity and acceleration.

We know that $\phi = ag \sin(kx - \sigma t)$ this we have derived this equation. The horizontal particle velocity or the orbital velocity u is given by $\frac{\partial \phi}{\partial x}$, definition of velocity potential, so, we get $ag \sin(kx - \sigma t)$ into $\cos(kx - \sigma t)$ + z by $\cos(kx - \sigma t)$ into $\sin(kx - \sigma t)$.

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- The horizontal water particle velocity or orbital velocity, u is given by

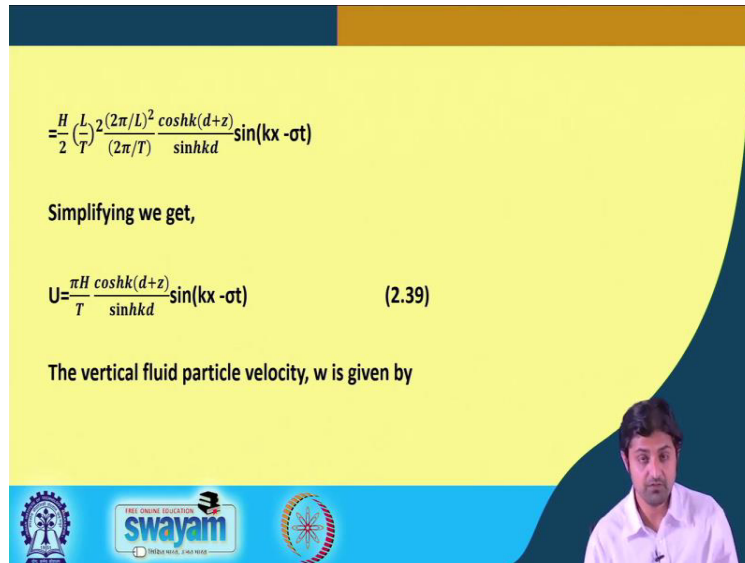
$$= \frac{ag}{\sigma} k \frac{\cosh k(d+z)}{\sinh kd} \cdot \tan hkd \cdot \sin(kx - \sigma t)$$
- Substituting the relationship $C^2 = \frac{g}{k} \tan hkd$ (2.30) and $a = \frac{H}{2}$
In the above expression we get

$$U = \frac{H}{2\sigma} C^2 k^2 \frac{\cosh k(d+z)}{\sinh kd} \sin(kx - \sigma t)$$

Whereas, so if we substitute the relationship $C^2 = g$ by $k \tan hkd$ and small a as H by 2 because this is wave height and this is wave celerity We get $u = \frac{H}{2} \sigma C^2 K$ squared divided by $\cos hkd$ represent by $\sin hkd$ into $\sin kx - \sigma t$ u it is enough if you remember the velocity potential equation because that you can differentiate and try to find out the velocities.

Maximum you will be given, find out the velocity that is that = 0 or at the water depth where is that = $-d$ because though that is pretty simple to calculate or maximum velocity when you talk about maximum velocity, this will be assumed to be one for the maximum velocity case.

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$$= \frac{H}{2} \left(\frac{L}{T} \right)^2 \frac{2(2\pi/L)^2 \cosh k(d+z)}{(2\pi/T) \sinh kd} \sin(kx - \sigma t)$$

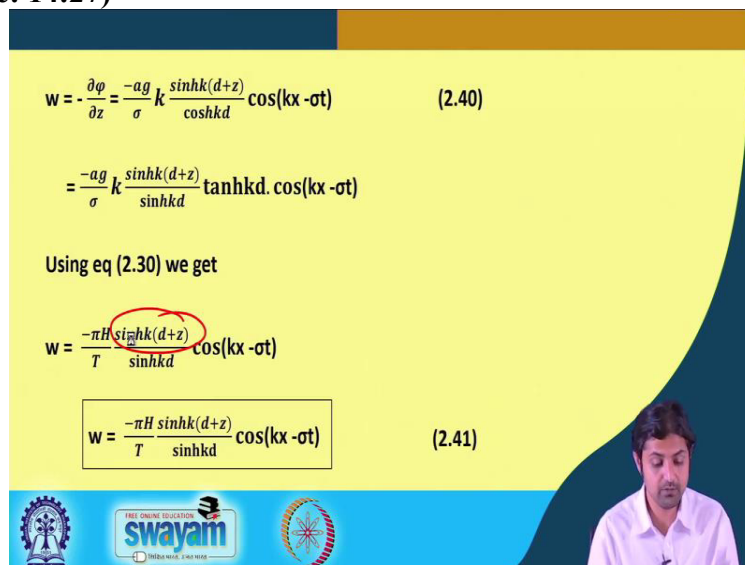
Simplifying we get,

$$U = \frac{\pi H \cosh k(d+z)}{T \sinh kd} \sin(kx - \sigma t) \quad (2.39)$$

The vertical fluid particle velocity, w is given by

So, just doing manipulation writing in terms of l and t this equation you can remember but not mandatory simplifying you will definitely remember the velocity potential equation and therefore, you can know derive this U even for your assignments and exams, so it is πH by $T \cos hkd + z$ divided by $\sin hkd$ into $\sin kx - \sigma t$, this is the periodic term periodic term in x and d $\sin kx - \sigma t$. And similarly we can find the vertical fluid velocity is given by $-\partial \phi / \partial z$.

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$$w = -\frac{\partial \phi}{\partial z} = -\frac{ag}{\sigma} k \frac{\sinh k(d+z)}{\cosh kd} \cos(kx - \sigma t) \quad (2.40)$$

$$= -\frac{ag}{\sigma} k \frac{\sinh k(d+z)}{\sinh kd} \tanh kd \cos(kx - \sigma t)$$

Using eq (2.30) we get

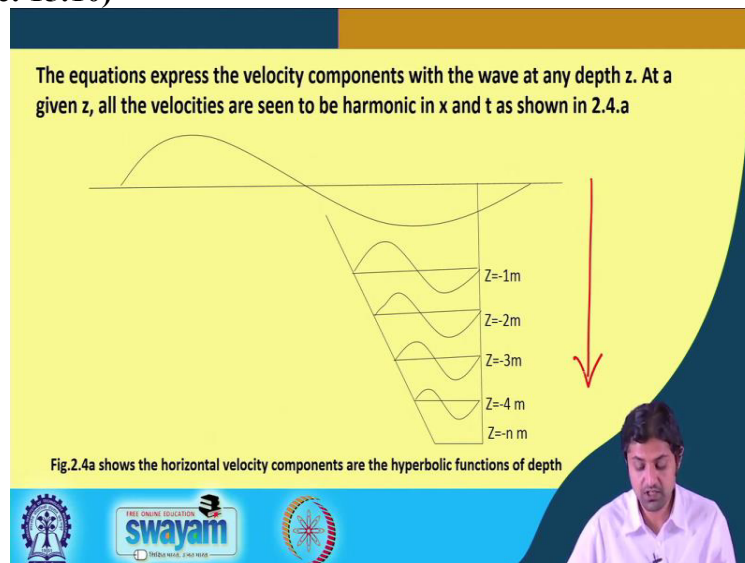
$$w = \frac{-\pi H \sinh k(d+z)}{T \sinh kd} \cos(kx - \sigma t)$$

$$w = \frac{-\pi H \sinh k(d+z)}{T \sinh kd} \cos(kx - \sigma t) \quad (2.41)$$

And therefore, we can you know similar substitutions will give us $-\pi H$ by $T \sinh k(d+z)$ divided by $\sinh kd$. Again, you do not have to remember this but if you do it is no harm because more

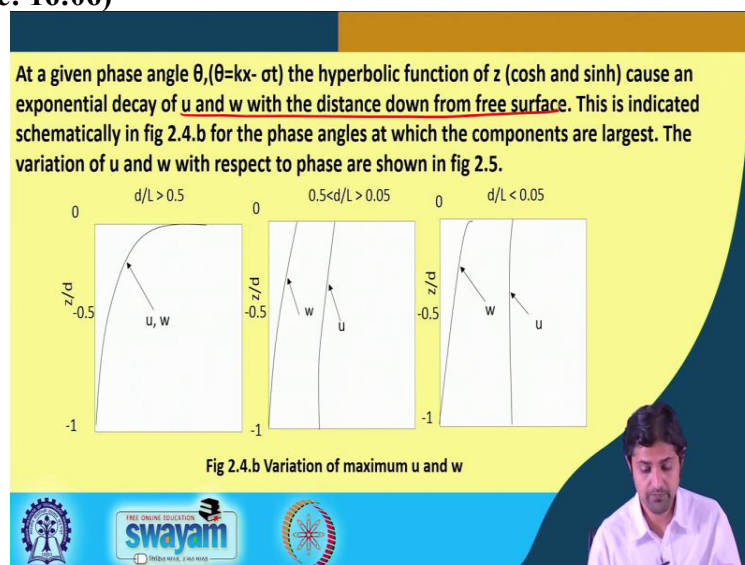
importantly it is the velocity potential that you must be remembering. So, w can return as $-\pi H$ by $T \sin hk$ into $d + z$ divided by $\sinh hk$ into $\cos kx - \sigma t$.

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So, the equation expresses the velocity components with the wave at any depth z at a given z all the velocities are seen to be harmonic in x and t as shown in figure the lets you know if we plot those its harmonic but with different said you know this is the variation which we are getting if we plot those velocities. So, figure 2.48 this figure it shows the horizontal velocity component or the hyperbolic functions of the depth you see $\sinh hk(d + z)$ hyperbolic functions of the pleasant and same with you also you see one is U is in $\cos hk$ and this is in $\sin hk$ we want w .

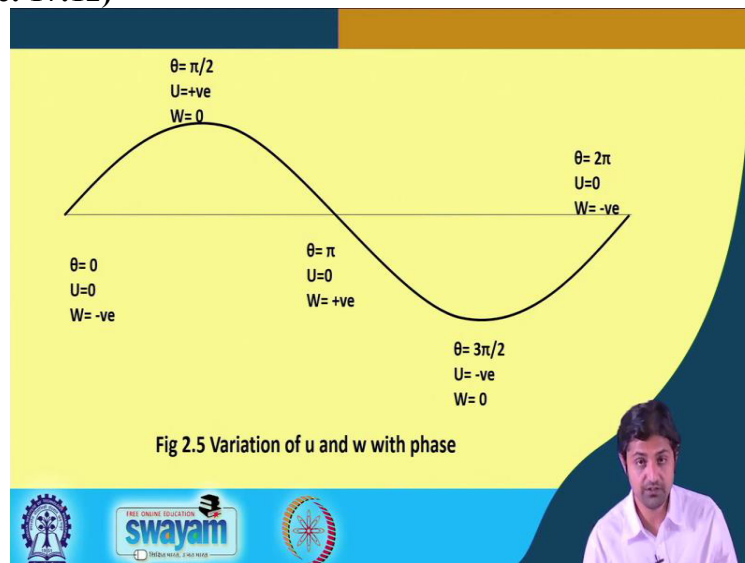
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We say at any phase angle θ phase angle is $kx - \sigma t$ the hyperbolic functions of z that is $\cosh$ and $\sinh$ cause an exponential decay of u and w with the distance down from the sea surface, this is indicated schematically in this figure. These are just the findings, you might you should not put a lot of effort and concentration and finding these values but, I mean this is pretty true right that there is an exponential decay of u and w with the distance down from the surface.

Variation of maximum u and w because cause and cause the you know if we take a phase angle fixed ϕ the whatever that is, that does not really matter because it will be the same for different z the periodic will be the same because $\cos \theta$ or $\sin \theta$ will remain the same.

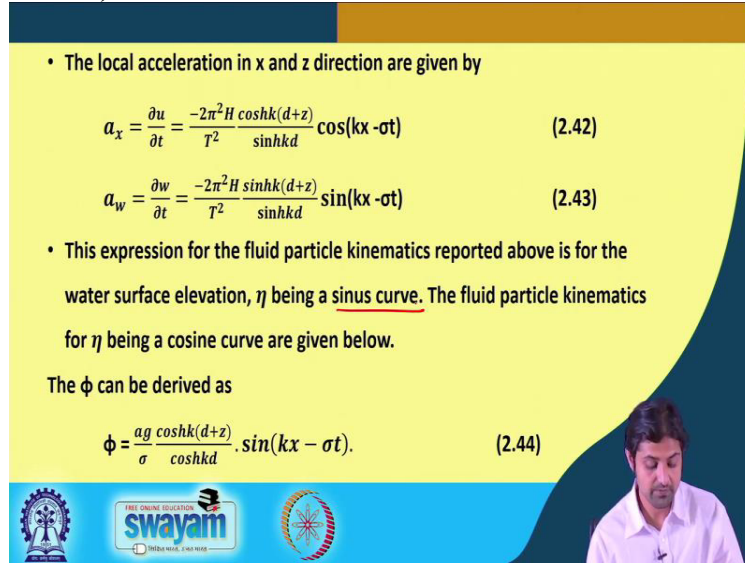
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So here the phase angle is deemed to be 0 $\theta = 0$, $u = 0$ and w is negative here θ is $\pi/2$ and u is assume u is positive and w is 0. At the top here $\theta = \pi$ u is 0, but w is positive. At $3\pi/2$ u is negative and $w = 0$. And if you put in the velocity potential and u and w equation, different θ , different values you are going to get and the sign will be determined from there so this is just a summary of those results obtained from this equation more importantly, this is $\theta = 0$ starting from here.

This is $\theta = \pi/2$ this is $\theta = \pi$ this θ will $3\pi/2$ and this is $\theta = 2\pi$. So, phase is changing by $\pi/2$ from this point to this point and to this point finally. So, this is a variation of u and w with phase.

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- The local acceleration in x and z direction are given by

$$a_x = \frac{\partial u}{\partial t} = \frac{-2\pi^2 H}{T^2} \frac{\cosh k(d+z)}{\sinh kd} \cos(kx - \sigma t) \quad (2.42)$$

$$a_w = \frac{\partial w}{\partial t} = \frac{-2\pi^2 H}{T^2} \frac{\sinh k(d+z)}{\sinh kd} \sin(kx - \sigma t) \quad (2.43)$$

- This expression for the fluid particle kinematics reported above is for the water surface elevation, η being a sinus curve. The fluid particle kinematics for η being a cosine curve are given below.

The ϕ can be derived as

$$\phi = \frac{ag}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin(kx - \sigma t). \quad (2.44)$$

Now, we have talked about the velocity similar way we should talk about acceleration as well. So, a_x is $\partial u / \partial t$ because u we have already obtained in terms of $\cosh$ and $\cos$. So, we simply differentiate u with respect to t . $\cos$ becomes $-\sin$ here with a negative sign. And similarly a_w will become $\sin$ with a negative sign, here it was signed, so it becomes $\cos$ and there was already a negative sign.

And in a_w this $\cos$ was then and it becomes $\sin$ after the differentiation with a negative sign so, this expression for fluid particle kinematic reported above is for water surface derivation being a sine curve. So, we started with a velocity potential where our time variation was $\sin kx - \sigma t$. So, similarly, if we assume a cosine curve, the velocities can be derived as $\sin kx - \sigma t$ correct.

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