Chapter 5

CABLES

5.1 Funicular Polygons

1 A cable is a slender flexible member with zero or negligible flexural stiffness, thus it can only transmit **tensile** forces[1].

2 The tensile force at any point acts in the direction of the tangent to the cable (as any other component will cause bending).

3 Its strength stems from its ability to undergo extensive changes in slope at the point of load application.

4 Cables resist vertical forces by undergoing **sag** (h) and thus developing tensile forces. The horizontal component of this force (H) is called **thrust**.

5 The distance between the cable supports is called the **chord**.

6 The sag to span ratio is denoted by

$$r = \frac{h}{l} \tag{5.1}$$

7 When a set of concentrated loads is applied to a cable of negligible weight, then the cable deflects into a series of linear segments and the resulting shape is called the **funicular polygon**.

8 If a cable supports vertical forces only, then the horizontal component H of the cable tension T remains constant.

■ **Example 5-1: Funicular Cable Structure**

Determine the reactions and the tensions for the cable structure shown below.

[1] Due to the zero flexural rigidity it will buckle under axial compressive forces.

$$= \frac{H}{\cos\theta_B} = \frac{51}{0.999} = \boxed{51.03 \text{ k}} \quad (5.6\text{-d})$$

$$T_{CD}; \qquad \tan\theta_C = \frac{4.6}{30} = 0.153 \Rightarrow \theta_C = 8.7\,\text{deg} \quad (5.6\text{-e})$$

$$= \frac{H}{\cos\theta_C} = \frac{51}{0.988} = \boxed{51.62 \text{ k}} \quad (5.6\text{-f})$$

■

5.2 Uniform Load

5.2.1 qdx; Parabola

9 Whereas the forces in a cable can be determined from statics alone, its configuration must be derived from its deformation. Let us consider a cable with distributed load $p(x)$ **per unit horizontal projection** of the cable length[2]. An infinitesimal portion of that cable can be assumed to be a straight line, Fig. 31.1 and in the absence of any horizontal load we have

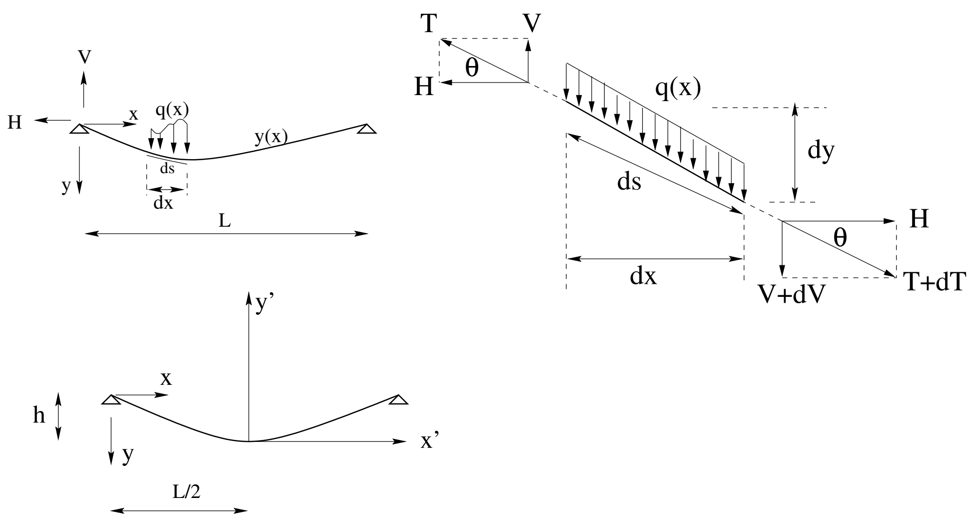

Figure 5.1: Cable Structure Subjected to $q(x)$

H =constant. Summation of the vertical forces yields

$$(+\downarrow)\, \Sigma F_y = 0 \Rightarrow -V + qdx + (V + dV) = 0 \quad (5.7\text{-a})$$

$$dV + qdx = 0 \quad (5.7\text{-b})$$

where V is the vertical component of the cable tension at x[3]. Because the cable must be tangent to T, we have

$$\tan\theta = \frac{V}{H} \quad (5.8)$$

[2] Thus neglecting the weight of the cable

[3] Note that if the cable was subjected to its own weight then we would have qds instead of pdx.

15 Combining Eq. 31.7 and 31.8 we obtain

$$y = \frac{4hx}{L^2}(L - x) \tag{5.18}$$

16 If we shift the origin to midspan, and reverse y, then

$$\boxed{y = \frac{4h}{L^2}x^2} \tag{5.19}$$

Thus the cable assumes a parabolic shape (as the moment diagram of the applied load).

17 The maximum tension occurs at the support where the vertical component is equal to $V = \frac{qL}{2}$ and the horizontal one to H, thus

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{\left(\frac{qL}{2}\right)^2 + H^2} = H\sqrt{1 + \left(\frac{qL/2}{H}\right)^2} \tag{5.20}$$

Combining this with Eq. 31.8 we obtain[5].

$$\boxed{T_{max} = H\sqrt{1 + 16r^2} \approx H(1 + 8r^2)} \tag{5.21}$$

5.2.2 † qds; Catenary

18 Let us consider now the case where the cable is subjected to its own weight (plus ice and wind if any). We would have to replace qdx by qds in Eq. 31.1-b

$$dV + qds = 0 \tag{5.22}$$

The differential equation for this new case will be derived exactly as before, but we substitute qdx by qds, thus Eq. 31.5 becomes

$$\boxed{\frac{d^2y}{dx^2} = -\frac{q}{H}\frac{ds}{dx}} \tag{5.23}$$

19 But $ds^2 = dx^2 + dy^2$, hence:

$$\frac{d^2y}{dx^2} = -\frac{q}{H}\sqrt{1 + \left(\frac{dy}{dx}\right)^2} \tag{5.24}$$

solution of this differential equation is considerably more complicated than for a parabola.

20 We let $dy/dx = p$, then

$$\frac{dp}{dx} = -\frac{q}{H}\sqrt{1 + p^2} \tag{5.25}$$

[5] Recalling that $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \cdot$ or $(1+b)^n = 1 + nb + \frac{n(n-1)b^2}{2!} + \frac{n(n-1)(n-2)b^3}{3!} + \cdots$; Thus for $b^2 << 1$, $\sqrt{1+b} = (1+b)^{\frac{1}{2}} \approx 1 + \frac{b}{2}$