Hydraulic Engineering
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Lecture # 55
Computational fluid dynamics (Contd.)

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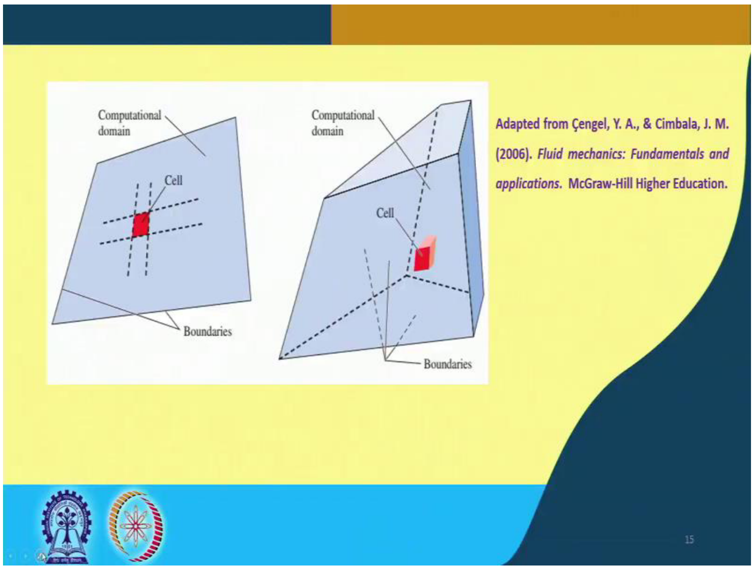

Welcome back students in this lecture, we will we are going to continue what we started in the last lecture with the grid generation. So, we finished that this slide. Therefore, we are going to proceed from this point onwards and see what further analysis can be done.

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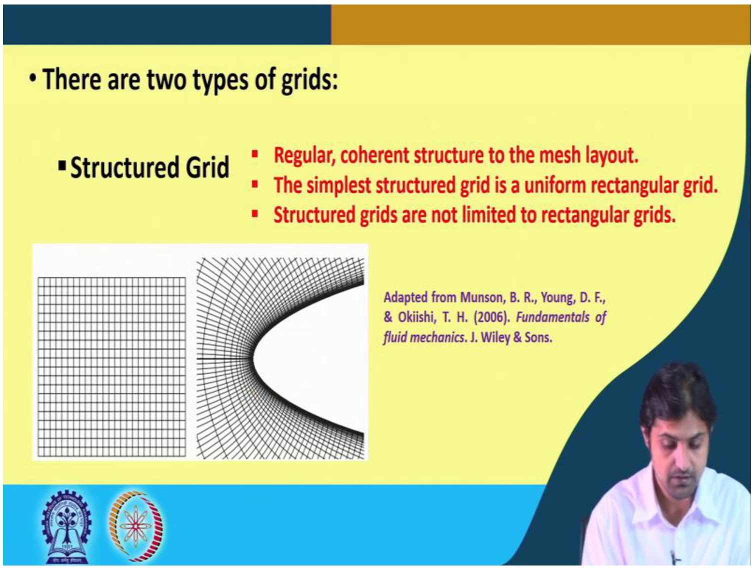

So, we talked about grids so, there are actually 2 types of grids. One is a structured grid and the structured grid means the grids are regular and coherent structure to the mesh layer. These are the simplest the structured grid and they are generally uniform rectangular grid those are called the structured grids. So, the structured grids look like this one here. So, you see they have a uniform rectangular grid. So these all the small shapes are rectangular.

Structured grids are not limited to rectangular grids only. So, these ones this could be off actually of any shape here it chose the rectangular grids only but you see these rectangular grids are decreasing in size.

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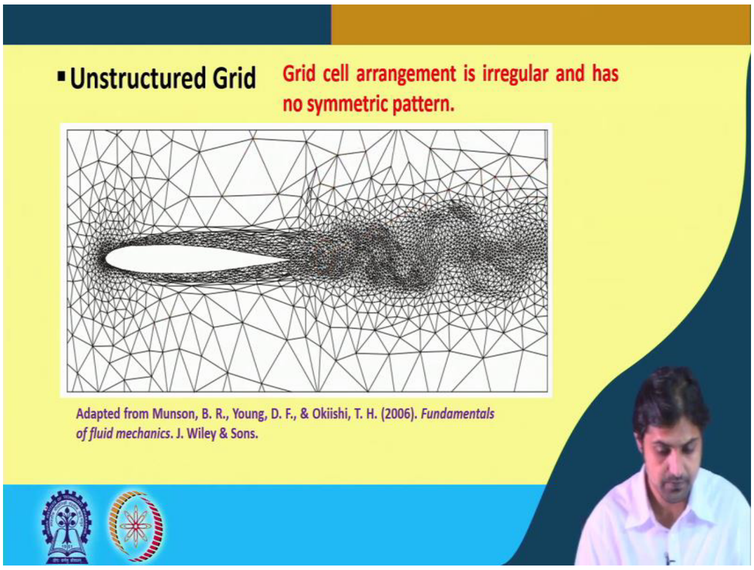

The second one are the unstructured grids. So, the grid cell arrangement is irregular and has no symmetry pattern if you see in the last one, there was a symmetry pattern here. If you consider this one you see symmetry cause this and here it is completely symmetry this one in the unstructured grids, the cell arrangement is irregular and has no symmetric pattern something like this, so, see the triangles, these are off, no specific, same type. You see this the cells here are very small, smallest smaller than these ones here largest.

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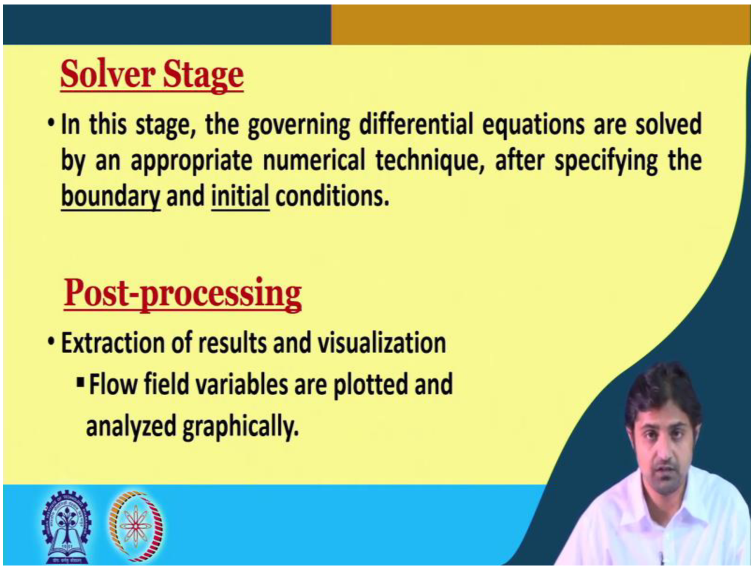

So, now after this regeneration we comes to the solver stage. So, in this stage that the governing differential equations are solved by an approximate numerical technique after specifying the boundary and the initial conditions. So, the actual real solution of those differential equations is done at this stage called the solver stage and in post processing, the extraction of results and visualization how the results appear is done. So, when it comes to extraction of results and visualization flow field variables are plotted and analyze graphically. So, this is the most common thing to do in the post processing.

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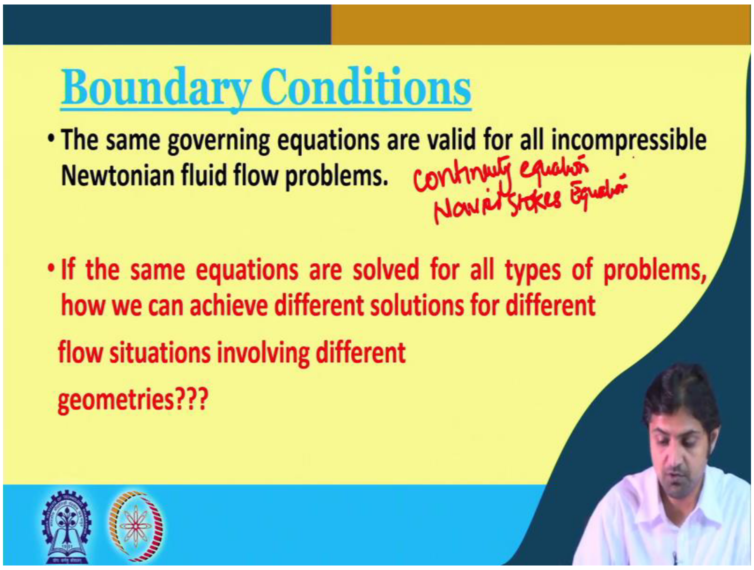

So, you use you know we mentioned about boundary and initial conditions and that is what we are going to discuss next. Boundary condition the same governing equations are valid for all compressible Newtonian fluid flow problems. So, if the same equations are solved for all types of problems, and how can we achieve different solutions for different flow situations involving different geometries that is the question because which governing equations are we going to talk we are talking about the continuity equation, Navier stokes equation.

So, this is common for all the fluid flow problems? Then how do we know and how do we achieve different solutions for different flow situations involving different geometries and this happens because of something called boundaries conditions.

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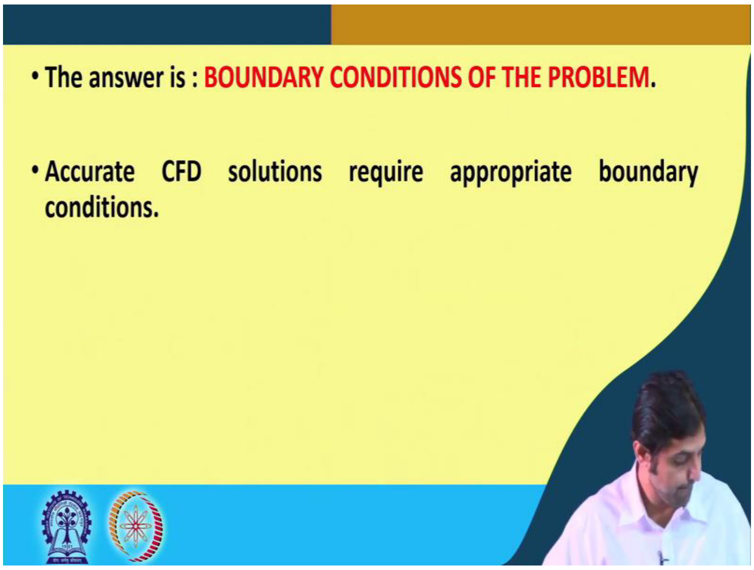

So the answer is boundary conditions of the problem. So, boundary conditions, I will tell you what that is for example, if the flow is occurring in a tank itself the velocity is 1 meters per second, this is one continuously occurring. So, this is 1 boundary condition, there could be other where you know the velocities could be 10 meters per second for example, so, these are different boundary conditions that is why we get different results.

Suppose, 1 of the boundary conditions could be this is closed at this end and the other one that it could be open. So, this is another boundary condition so, because of this difference in boundary conditions we have different solutions of the problem.

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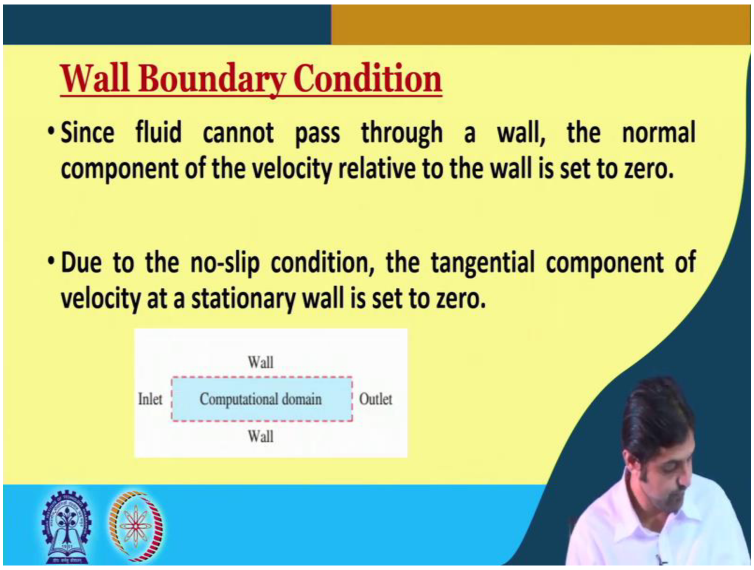

Now, as I said I mentioned about being closed and you know being open there is a wall at the boundary these wall and other concepts you have read in your previous lectures of hydraulic engineering. So, we are going to talk about the wall boundary condition, since fluid cannot pass through a wall the normal component of the velocity relative to the wall is set to 0 and this is what is this called this is called no slip condition. Therefore, due to the no slip condition the tangential component of the velocity at a stationary wall is set to 0.

So, you see there if there is an inlet here, there is a wall here there is an outlet. So, the velocity at this point does not matter what the velocity here is at this particular point anywhere around across this wall the velocity will be 0 due to no slip condition.

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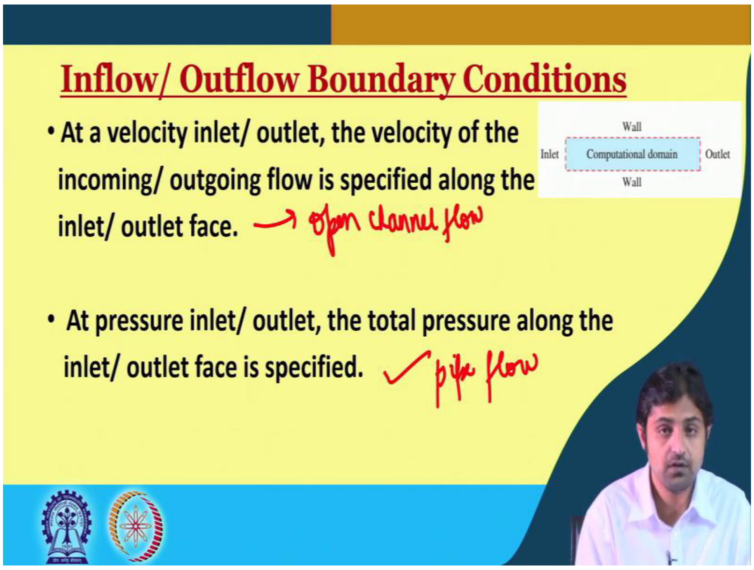

No not talking about the inflow and the outflow boundary conditions. So, you see this is inlet this is outlet. So, the here from here the inflow will be there, and from here out flow will be there. In means coming in and out means going out. So, at a velocity inlet or outlet the velocity of the incoming or outgoing flow specified along the inlet outlet phase at pressure inlet and outlet the total pressure along the inlet and outlet phase specified.

So, the in the inlet and outlet can be the specification can be in 2 forms, whether if whether we want to specify the inlet flow velocities at the inlet or outlet or the pressure. 1 typical example here is the for the pressure is the pipe flow. Here it is open channel flow.

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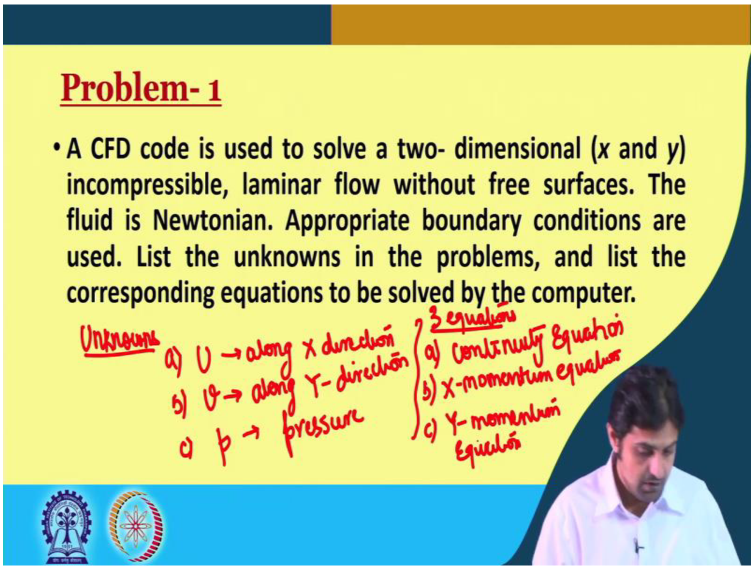

So, there is one question which I would want to know, solve solver discuss. So there is a CFD code which is used to solve a 2 dimensional in 2 dimension X and Y incompressible laminar flow without free surfaces. The fluid is Newtonian. So appropriate boundary conditions are used. So the question here is now lists the unknowns in the problem and list the corresponding equations to be solved by the computer.

So first important information that we have is that it is 2 dimensional in nature. So, the unknowns, you can start guessing the unknowns first is going to be velocity you which is along X direction. Secondly, the velocity V, which is along Y direction. The third one is going to be since there is no more third dimension there is not going to be a W direct W velocity, but there definitely will be p that is pressure and how are we going to solve these which equations are we going to solve for U V and P.

We are going to use 3 we see there are 3 equations. So, the first equation is very common conservation equation which is the continuity equation. The second one will be the X momentum equation so momentum equation in X direction. We will have a Y momentum equation. So, this is very quiet simple to you know imagine or guess that in 2 dimension only 2 dimensional velocities and the pressure is going to be the unknown and therefore, to solve we just need the three equations continuity, x momentum and y momentum equations.

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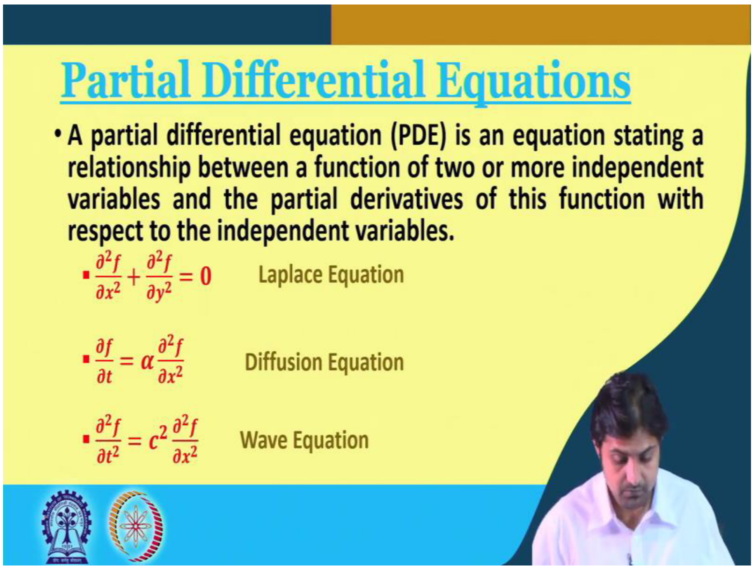

So, now, we come to the, in the solver stage we come to we have this way we have mentioned about the boundary condition and now we come to discuss about the partial differential equations. So, partial differential equation PDE is an equation stating a relationship between a function of two or more independent variables and the partial derivatives of this function with respect to the independent variable. So, this is the definition of partial differential equation.

So, it states it states the relationship between a function of two or more independent variables and the partial derivatives of this function with respect to the independent variable. For example, this equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

. So, these are independent variables x and y and f is an independent variable and this equation is called Laplace equation from your previous experience you should be knowing this another equation is

$$\frac{\partial f}{\partial t} = \alpha \frac{\partial^2 f}{\partial x^2}$$

So, this is a diffusion equation the third equation which we have written is

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

and this equation is a wave equation. So, these are the 3 most common types of equation in partial differential equation that we know.

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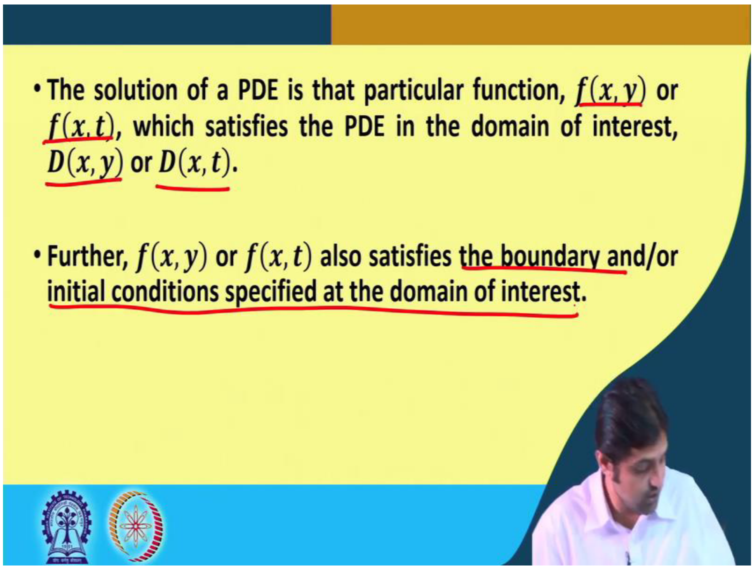

So, this is solution of the partial differential equation is that particular function f x y or f x, t, which satisfies the partial differential equation in the domain of interest domain is given by D(x y) or D (x, t), f(x, y) is when the function is based only on x and y, f(x, t) when if it is dependent on only x and t. Further f (x, y) or f (x, t). These are 2 different type of partial differential equations f x, y or f x, t they also satisfy the boundary and our initial conditions specified at the domain of interest. So, they must be satisfying they must be satisfied at all places in the domain or and at all times for which the calculation is being done.

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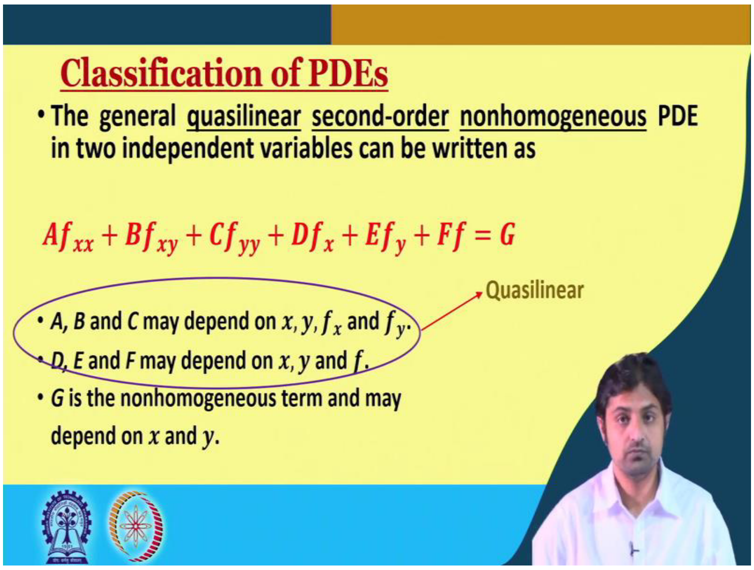

So, going to take a slight detour, because it is important that we try to remember what the classification of partial differential equations look like. So, they are general quasilinear second order non-homogeneous partial differential equation in 2 independent variables can be written as Af xx + Bf xy + Cf yy + Df x + Ef y + Ff = G, this is a general quasilinear second order we have x x, x y and y y that is second order non-homogeneous or partial differential equation.

So, A, B and C may depend on x, y, f x and f y, D, E and F may depend on x y and f. So, this is this means quasilinear. G is non-homogeneous term and may depend on x and y that is why we read non-homogeneous this is not equal to 0.

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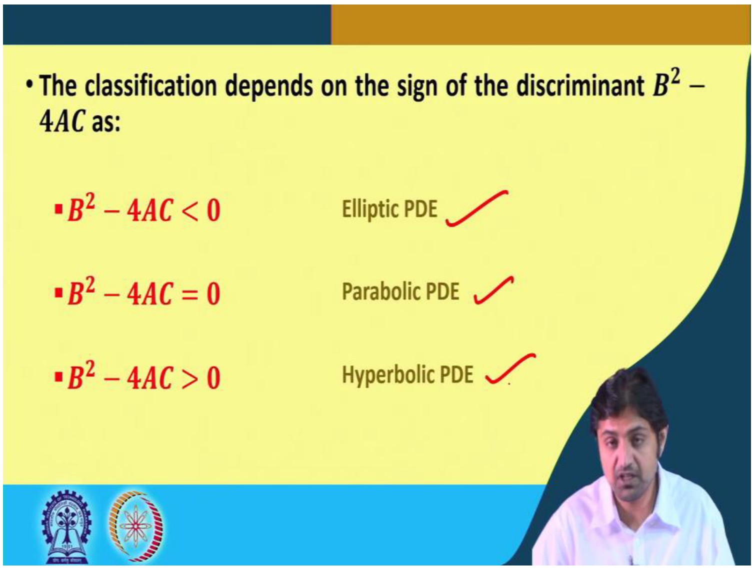

So, the classification depends on the sign of the discriminant B^2- 4AC as so, in this particular equation Af xx + Bf xy + Cf yy + Df x + Ef y + F f = G, the solution will depend on the value of B^2- 4AC. And how does it depend if B^2- 4AC is less than 0, then it is going to be elliptical partial differential equation. If B^2- 4AC = 0 is going to be a parabolic partial differential equation, if B^2- 4AC is greater than 0 then it is going to be a hyperbolic partial differential equation. You this you can recall from your mathematics class.

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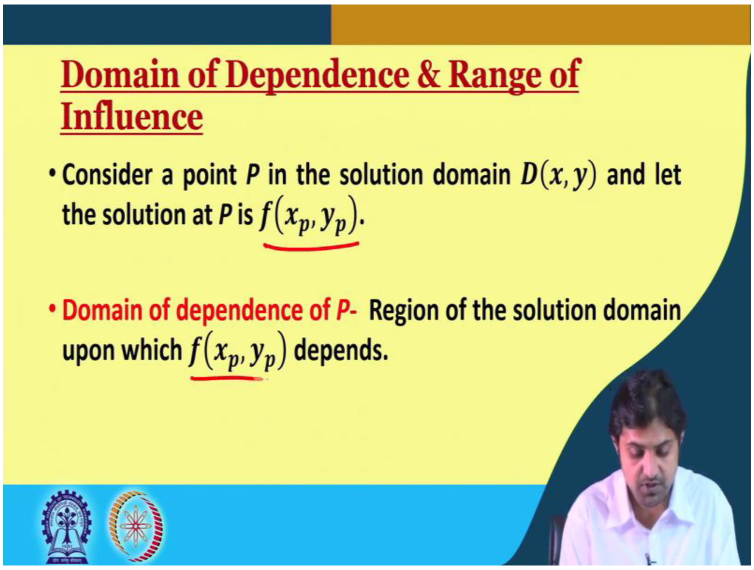

Now going to domain of dependence and range of influence. So, if you consider a point P in the solution domain, so, there is a point in the domain P and let the solution at P if we assume that the solution at that particular point P is f (x p, y p) we assume that then the domain of dependence of P is the region of solution domain upon which f(x p, y p) depends.

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