

- From Eq. 29 and Eq. 32, we get

$$\frac{V_{avg}}{u_*} = 5.75 \log_{10} \frac{R}{k} + 4.75 \quad (\text{Eq. 33}) \quad \checkmark$$

So, what is this? This is the average velocity divided by the frictional velocity for the turbulent pipe flow case.

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- From the above equations, we can write

$$\frac{u - V_{avg}}{u_*} = 5.75 \log_{10} \left[\frac{\frac{u_* y}{\nu}}{\frac{u_* R}{\nu}} \right] + 5.5 - 1.75$$

or

$$\frac{u - V_{avg}}{u_*} = 5.75 \log_{10} \left[\frac{y}{R} \right] + 3.75 \quad (\text{Eq. 34}) \quad \checkmark$$

Now, the difference of the velocity at any point and the average velocity for smooth pipes. For smooth pipes, we have seen that the u by u star we had, we came, we derived this equation, correct. And we also just now we saw that V average by u star is going is this equation and therefore, from the above equation we can simply subtract these two equation and we are able to find u minus V average by u star. So, we do this equation, this minus this, this minus this. So, it will be u star is common, so, it will be u minus V average by u star is equal to $5.75 \log$ to the base 10 u star y by νu star R by ν .

So, ν and ν can get cancelled, it will be y by R or we can say, u minus V average by u star is equal to $5.75 \log y$ by R plus 3.75 . So, this is an important equation again. So, all these you either you remember or you can actually derive it. It is very simple, starting from the, starting from the basic logarithmic velocity profile.

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□ **For Rough Pipes**

- For rough pipes


$$\frac{u}{u_*} = 5.75 \log_{10} \left(\frac{y}{k} \right) + 8.5 \quad \checkmark$$

and

$$\frac{V_{avg}}{u_*} = 5.75 \log_{10} \frac{R}{k} + 4.75 \quad \checkmark$$

- Utilizing the above equations:

$$\frac{u - V_{avg}}{u_*} = 5.75 \log_{10} \left[\frac{y/k}{R/k} \right] + 8.5 - 4.75$$



Now, we did it for smooth pipes. Now, for rough pipes, we have an equation and we also obtained V average by u star. We do the same procedure, we subtract this equation, we subtract this equation from this equation. So, this minus this and utilizing the above equation, we get u minus V average by u star. So, we have got y by k divided by R by k plus 8.5 minus 4.75

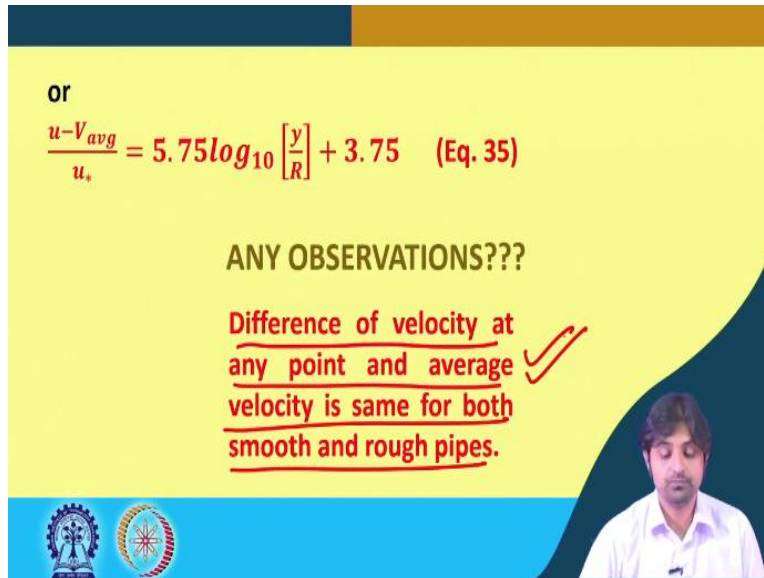
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or

$$\frac{u - V_{avg}}{u_*} = 5.75 \log_{10} \left[\frac{y}{R} \right] + 3.75 \quad (\text{Eq. 35})$$

ANY OBSERVATIONS???

Difference of velocity at any point and average velocity is same for both smooth and rough pipes. ✓



Or we can simply get $u - V_{avg}$ by u_* is equal to $5.75 \log_{10} y$ by R plus 3.75. Is there any observation? If you see for either for smooth or for rough this difference came out to be the same. So, the observation is the difference of velocity at any point and average velocity is same for both smooth and rough pipes. This is important to know that.

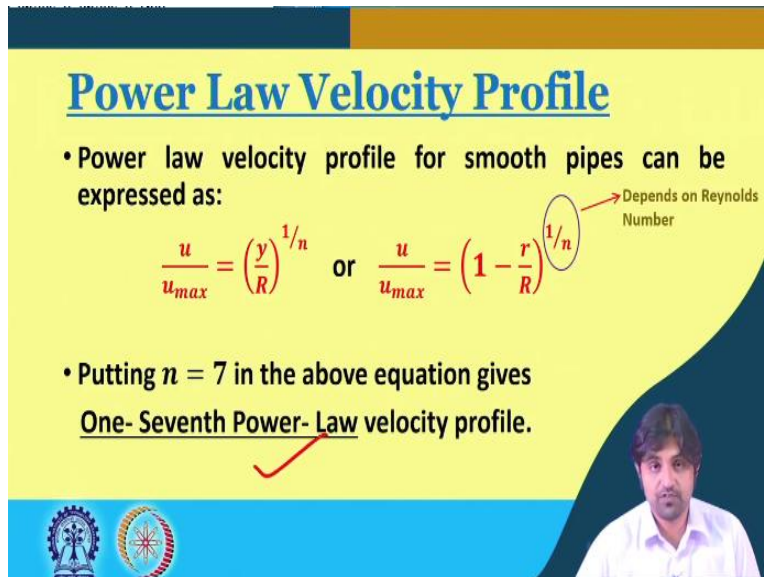
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Power Law Velocity Profile

- Power law velocity profile for smooth pipes can be expressed as:

$$\frac{u}{u_{max}} = \left(\frac{y}{R} \right)^{1/n} \quad \text{or} \quad \frac{u}{u_{max}} = \left(1 - \frac{r}{R} \right)^{1/n}$$

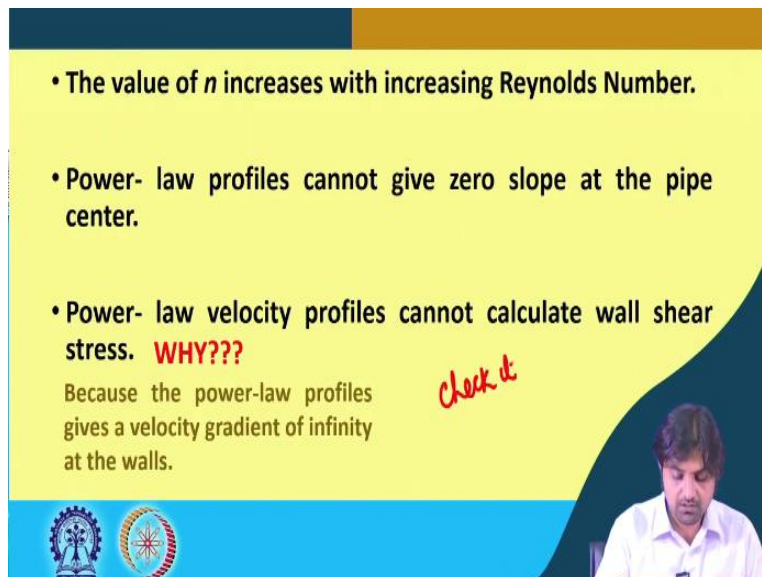
Depends on Reynolds Number
- Putting $n = 7$ in the above equation gives One-Seventh Power-Law velocity profile. ✓



Now, about the power law velocity profile, a little bit on that. So, the power law velocity profile for smooth pipes can be expressed as, it is, u by u_{max} can be written as y by R to the power 1 by n . This is the power law velocity that was given or we can always write in terms of R because y is, y is R minus small r and that you substitute here, you will end up in this equation. So, this $1/n$

depends upon the Reynolds number, putting n is equal to 7 in above equation gives one-seventh power law velocity profile. This is one of the very famous velocity profiles.

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• The value of n increases with increasing Reynolds Number.

• Power-law profiles cannot give zero slope at the pipe center.

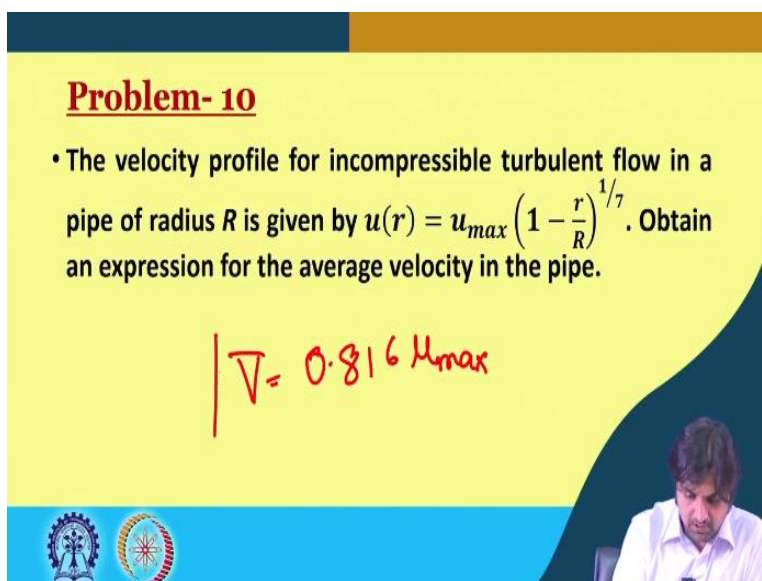
• Power-law velocity profiles cannot calculate wall shear stress. **WHY???**

Because the power-law profiles gives a velocity gradient of infinity at the walls. *check it*

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The value of n will increase with increasing Reynolds number. Power law velocity profiles cannot give 0 slope at the pipe center. So, power law of velocity profiles also cannot calculate the wall shear stress. Why? Because the power law profiles gives a velocity gradient of infinity at the walls. That you can try, by substituting r is equal to, small r is equal to capital R . You can check it check it at.

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Problem- 10

• The velocity profile for incompressible turbulent flow in a pipe of radius R is given by $u(r) = u_{max} \left(1 - \frac{r}{R}\right)^{1/7}$. Obtain an expression for the average velocity in the pipe.

$V = 0.816 u_{max}$

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Now, we are going to solve one of the problems. So, the velocity profile for incompressible turbulent fluid in a pipe of radius R is given by u of r as u_{\max} into 1 minus r by R to the power 1 by 7 . Obtain an expression for the average velocity in the pipe. Hear by chance, or by, you know, you get 1 by 7 power law. So, how to attack this type of problems and actually you can be given any such profile and you should follow the same procedure as I am going to do now, so that, you are able to calculate the average velocity in the pipe. So, how to do that? We simply go to white screen.

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Given $u(r) = u_{\max} \left(1 - \frac{r}{R}\right)^{1/7}$

$$\bar{V} = \frac{1}{\pi R^2} \int_0^R u(r) 2\pi r dr = \frac{1}{\pi R^2} \int_0^R u_{\max} \left(1 - \frac{r}{R}\right)^{1/7} 2\pi r dr$$

$$\bar{V} = \frac{2u_{\max}}{R^2} \int_0^R \left(1 - \frac{r}{R}\right)^{1/7} r dr$$

$$\bar{V} = 2u_{\max} \int_0^R \left(1 - \frac{r}{R}\right)^{1/7} \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \quad \text{--- ①}$$

Let $1 - \frac{r}{R} = x$

$$\Rightarrow dx = -d\left(\frac{r}{R}\right)$$

And we write given, u of r is given as u_{\max} into 1 minus r by R to the power 1 by 7 . So, the average velocity profile will be simply 1 pi divided by the entire area 0 to R u r 2 pi r dr or we can write, 1 by pi R square integral 0 to R u_{\max} into 1 minus r by R to the power 1 by 7 2 pi r dr . Therefore, we can take out u_{\max} outside and we can also take 2 pi also outside.

So, this pi, pi gets cancelled, 2 comes out, so, it became $2 u_{\max}$ by R square because pi gets cancelled integral 0 to R 1 minus r by R to the power 7 r dr or what we can do is, \bar{V} bar is $2 u_{\max}$ integral 0 to R , we will keep this one as 1 by 7 , and we can write r by R and $d r$ by R and call it one. Let, 1 minus r by R as x , this means, dx is equal to minus $d r$ by R . And so, if we substitute this, in this equation, what we can do? I will start from,

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$$\bar{V} = 2u_{\max} \int_0^1 x^{1/7} (1-x) dx$$

$$\bar{V} = 2u_{\max} \int_0^1 x^{1/7} (x-1) dx$$

$$= 2u_{\max} \int_0^1 (x^{8/7} - x^{1/7}) dx$$

$$= 2u_{\max} \left[\frac{7}{15} x^{15/7} - \frac{7}{8} x^{8/7} \right]_0^1$$

$$\bar{V} = 2u_{\max} \left[\frac{7}{8} - \frac{7}{15} \right]$$

$$\bar{V} = 2u_{\max} \left[\frac{7}{8} - \frac{7}{15} \right]$$

$$\bar{V} = 0.816 u_{\max}$$

So, using the, we write \bar{V} is equal to $2 u_{\max}$ integral x to the power 1 by 7 one minus x minus dx . So, that is from 1 to 0 . So, it will be $2 u_{\max}$ integral 0 to 1 x to the power 1 by 7 x minus 1 dx or $2 u_{\max}$ x to the power 8 by 7 minus x to the power 7 dx , that is, 1 to 0 . These limits have to be checked 1 to 0 . So, it was 1 to 0 , 1 to 0 , 1 to 0 .

So, \bar{V} prime can be $2 u_{\max}$ into $7/15$ x to the power 15 by 7 minus $7/8$ x to the power $8/7$, the limits from 1 to 0 for both side. So, \bar{V} is $2 u_{\max}$ and we substitute one as 1 and one as 0 . So, it will be 7 by 8 minus $7/15$. So, \bar{V} will be written by $2 u_{\max}$ into, just copying the same thing from the last line, $7/15$ and \bar{V} is going to be $0.816 u_{\max}$. So, this is actually a general way to calculate the average velocity. So, does not matter what the profile is, this is how you are going to proceed. So, an expression I will write it down here as well is $0.816 u_{\max}$.

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So, these are the references, as I have already shown you in the book, I mean, also in the introduction slides. And so, this week's lecture on the laminar and turbulent flow is finished. I will see you next week with another set of lectures on hydraulic engineering. Thank you so much. Have a nice weekend.