LECTURE 24

Terzaghi's 1D Consolidation Equation

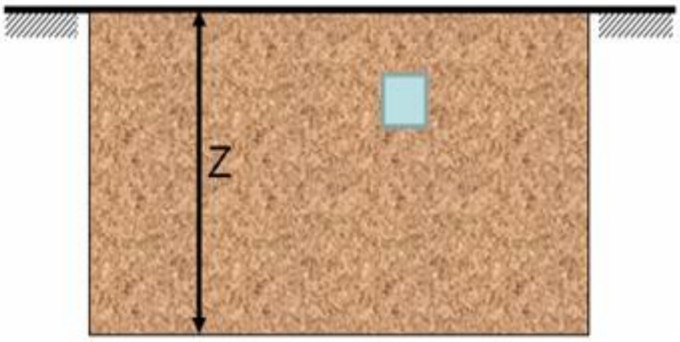


Figure Saturated soil Strata

Assumptions:

- ✓ The soil medium is completely saturated
- ✓ The soil medium is isotropic and homogeneous
- ✓ Darcy's law is valid for flow of water
- ✓ Flow is one dimensional in the vertical direction
- ✓ The coefficient of permeability is constant
- ✓ The coefficient of volume compressibility is constant
- ✓ The increase in stress on the compressible soil deposit is constant
- ✓ Soil particles and water are incompressible

One dimensional theory is based on the following hypothesis

1. The change in volume of soil is equal to volume of pore water expelled.

2. The volume of pore water expelled is equal to change in volume of voids.

3. Since compression is in one direction the change in volume is equal to change in height.

The increase in vertical stress at any depth is equal to the decrease in excess pore water pressure at the depth

$$\Delta\sigma' = \Delta u$$

This is Terzaghi's one dimensional consolidation equation

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

This equation describes the variation of excess pore water pressure with time and depth

Limitation of 1D consolidation

1. In the deviation of 1D equation the permeability (Kz) and coefficient of volume compressibility (mv) are assumed constant, but as consolidation progresses void spaces decrease and this results in decrease of permeability and

therefore permeability is not constant. The coefficient of volume compressibility also changes with stress level. Therefore Cv is not constant

2. The flow is assumed to be 1D but in reality flow is three dimensional

3. The application of external load is assumed to produce excess pore water pressure over the entire soil stratum but in some cases the excess pore water pressure does not develop over the entire clay stratum.

Solution of 1D consolidation

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \quad \text{----------------} \; 1$$

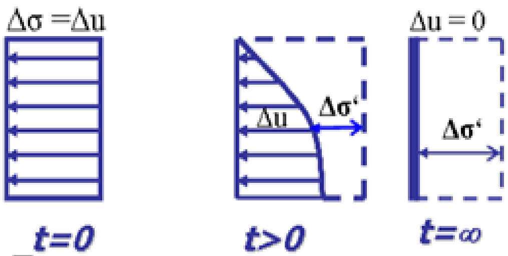
The solution of variation of excess pore water pressure with depth and time can be obtained for various initial conditions.

Uniform excess pore water pressure with depth

1. Single Drainage (Drainage at top and bottom impervious)

2. Double Drainage (Drainage at top and bottom)

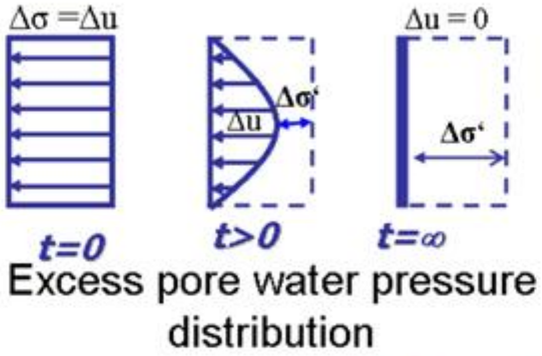
Single Drainage (drainage at top and bottom impervious)



Excess pore water pressure distribution

Excess porewater pressure distribution of single drainage

Double Drainage



Excess porewater pressure distribution of double drainage

Boundary Conditions are

i) At t = 0 $\Delta u = \Delta\sigma$ and $\Delta\sigma' = 0$

ii) At the top z = 0 $\Delta u = 0$ $\Delta\sigma = \Delta\sigma'$

iii) At the bottom z = 2Hdr $\Delta u = 0$ $\Delta\sigma = \Delta\sigma'$

A solution of equation (1) for the above boundary conditions using Fourier series is given by

$$\Delta u_{(z,t)} = \sum_{m=0}^{\infty} \frac{2\Delta u_0}{M} \sin\left(\frac{MZ}{H_{dr}}\right) e^{-M^2 T_v}$$

$M = \frac{\pi}{2}(2m+1)$ Where m = +ve integer with values from 0 to ∞

$Tv = \frac{c_v t}{H_{dr}^{\ 2}}$ Where Tv =Time factor (dimensionless)

Graphical solution of 1D consolidation equation

$$\Delta u_{(z,t)} = \sum_{m=0}^{\infty} \frac{2\Delta u_0}{M} \sin\left(\frac{MZ}{H}\right) e^{-M^2 T_v}$$

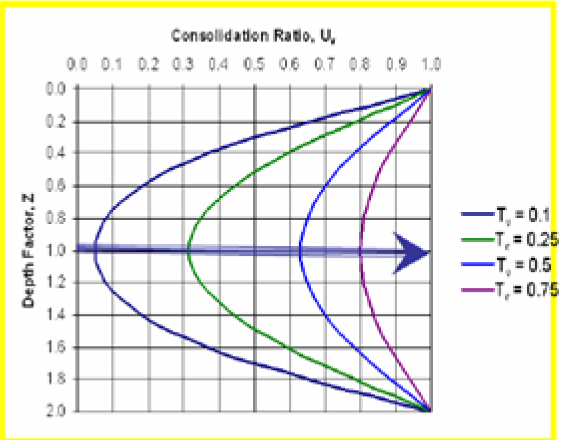
The solution of consolidation equation consists of the following three variables

1. The depth z
2. The excess pore water pressure _u
3. The time (t) after application of loading

The above variables are expressed in the form of the following non-dimensional terms as

Sl. No	Variables	Non-dimensional terms
1	Depth (z)	Z = z /H (Drainage path ratio)
2	Excess pore pressure (Δu)	Uz --- consolidation ratio This represents the dissipated pore water pressure to initial excess pore water pressure
3	Time (t)	T_v (Time factor)

The graphical solution of the above equation is as shown below



Terzaghi's solution for one-dimentional consolidation

This indicates the progress of consolidation with time and depth for a given set of boundary conditions.

Degree of Consolidation (U_z)

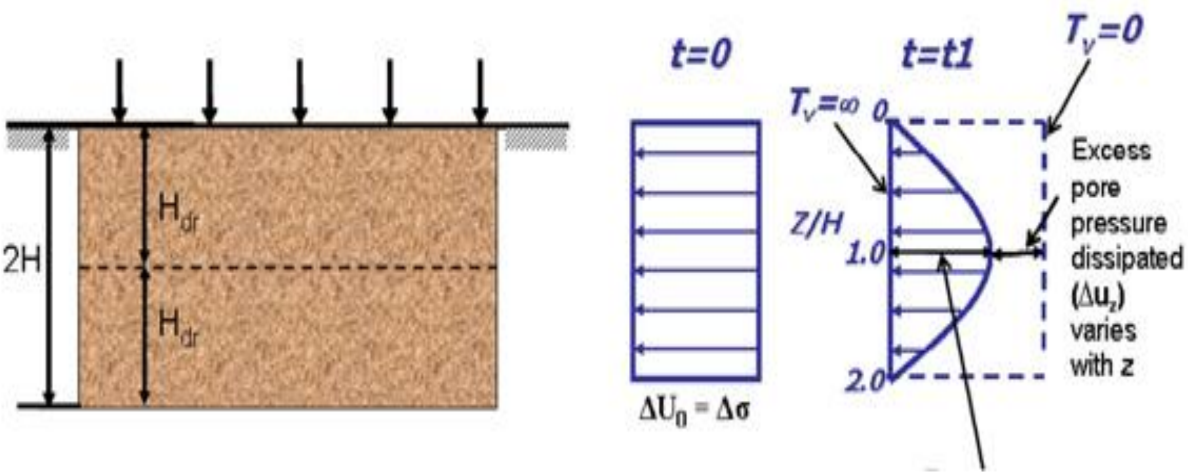


Figure: Selection of clay ayer and Excess pore water pressure distribution

The degree of consolidation at any depth is given by

$$U_z = \frac{\Delta u_0 - \Delta u_z}{\Delta u_0}$$

$$1 - \frac{\Delta u_z}{\Delta u_0} = \frac{\Delta \sigma'_z}{\Delta u_0}$$

Δu_o = Initial excess pore water pressure at that depth

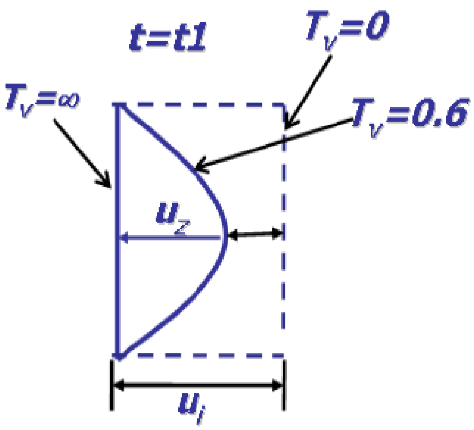
Δu_z = Excess pore water pressure at that depth

$$u_z = 1 - \frac{\Delta u_z}{\Delta u_0}$$

$$u_z = 1 - \sum_{m=0}^{\infty} \frac{2}{M} \sin\left(\frac{MZ}{H}\right) e^{-M^2 T_v}$$

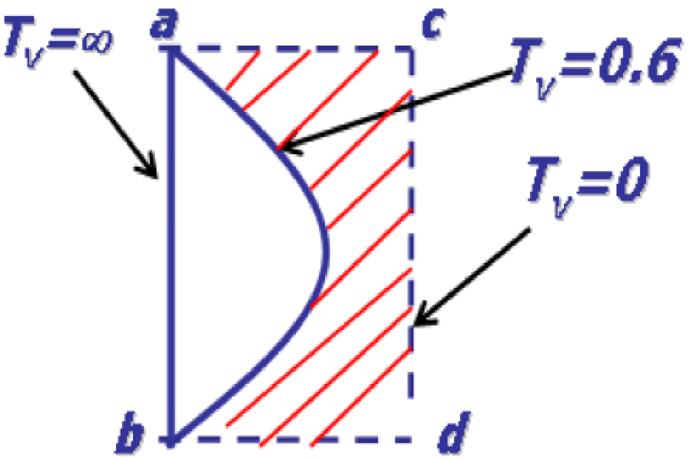
u_z= Degree of consolidation at a particular depth at any given time

From practical point of view, the average degree of consolidation over the entire depth at any given time is desirable. At any given time $\boldsymbol{u_z}$ *varies* with location and hence the degree of consolidation also varies.



u_z = Degree of consolidation at a certain level

Average Degree of Consolidation (U)



The average degree of consolidation for the whole soil deposit at any time is given by

$$U = \frac{\text{Area of the diagram of excess pore water pressure dissipated at any time}}{\text{Area of the diagram of initial excess pore water pressure}}$$

$$U = \frac{\text{Area Shaded}}{\text{Area of abcd}}$$

Mathematically $U = f(T_v)$

Consolidation of Soils

$$u = 1 - \sum_{m=o}^{\infty} \frac{2}{M} e^{-M^2 T_v}$$

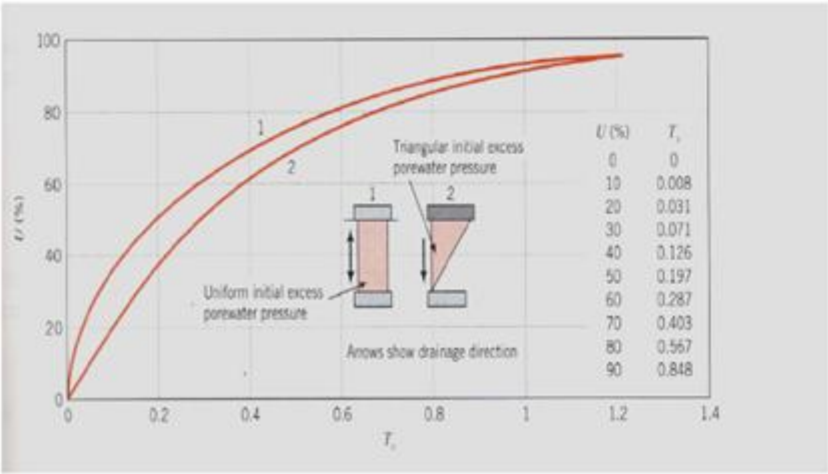


Figure: Degree of Consolidation versus time factor (T_v)

As per Taylor (1948) solution, the following approximation is possible

$$when \quad U \leq 60\,\% \qquad Tv = \frac{\pi}{4} U^2$$

$$For \quad U > 60\% \qquad Tv = 1.781 - 0.933 \log(100 - U\%)$$

Typical values of T_v

U = 50%	Tv = 0.197
U = 60%	Tv = 0.287
U = 90%	Tv = 0.848