Chapter 20

BRACED ROLLED STEEL BEAMS

1 This chapter deals with the behavior and design of **laterally supported** steel beams according to the LRFD provisions.

2 If a beam is not laterally supported, we will have a failure mode governed by lateral torsional buckling.

3 By the end of this lecture you should be able to select the most efficient section (light weight with adequate strength) for a given bending moment and also be able to determine the flexural strength of a given beam.

20.1 Review from Strength of Materials

20.1.1 Flexure

4 Fig.20.1 shows portion of an originally straight beam which has been bent to the radius ρ by end couples M, thus the segment is subjected to pure bending. It is assumed that plane cross-sections normal to the length of the unbent beam remain plane after the beam is bent. Therefore, considering the cross-sections AB and CD a unit distance apart, the similar sectors Oab and bcd give

$$\varepsilon = \frac{y}{\rho} \tag{20.1}$$

where y is measured from the axis of rotation (neutral axis). Thus strains are proportional to the distance from the neutral axis.

5 The corresponding variation in stress over the cross-section is given by the stress-strain diagram of the material rotated 90^o from the conventional orientation, provided the strain axis ε is scaled with the distance y (Fig.20.1). The bending moment M is given by

$$M = \int y\sigma dA \tag{20.2}$$

where dA is an differential area a distance y (Fig.20.1) from the neutral axis. Thus the moment M can be determined if the stress-strain relation is known.

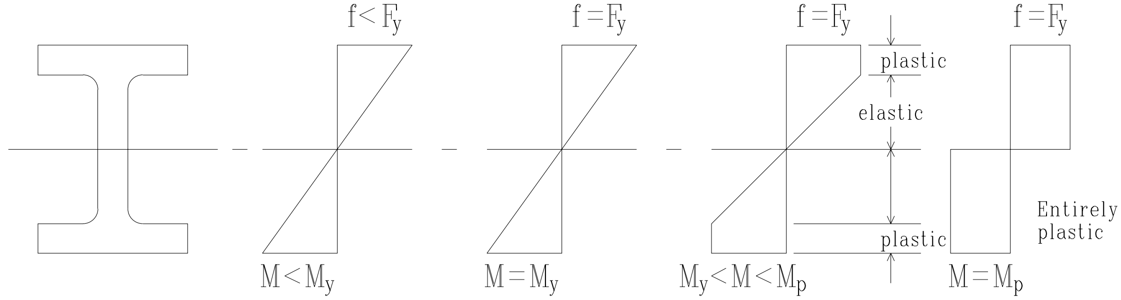

Figure 20.2: Stress distribution at different stages of loading

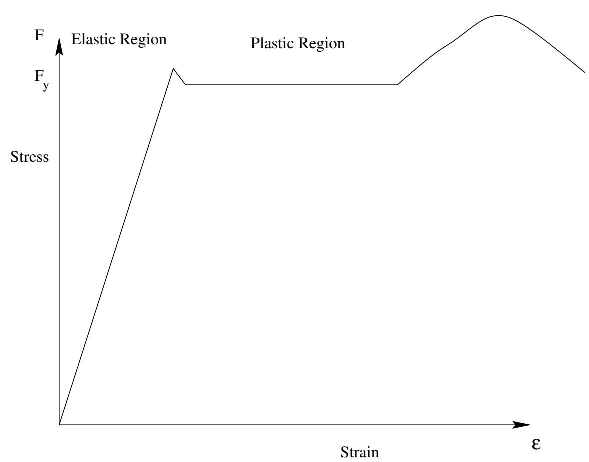

Figure 20.3: Stress-strain diagram for most structural steels

$$
\begin{aligned}
F_2 &= \int_A \frac{(M+dM)y}{I} dA && (20.8\text{-c}) \\
\tau b dx &= F_2 - F_1 && (20.8\text{-d}) \\
&= \int_A \frac{dMy}{I} dA && (20.8\text{-e}) \\
\tau &= \frac{dM}{dx}\left(\frac{1}{Ib}\right)\int_A y dA && (20.8\text{-f}) \\
& && (20.8\text{-g})
\end{aligned}
$$

substituting $V = dM/dx$ we now obtain

$$
\boxed{\begin{array}{lcl} \tau & = & \frac{VQ}{Ib} \\ Q & = & \int_A y dA \end{array}} \qquad (20.9)
$$

this is the shear formula, and Q is the first moment of the area.

12 For a rectangular beam, we will have a parabolic shear stress distribution.

13 For a W section, it can be easily shown that about 95% of the shear force is carried by the web, and that the average shear stress is within 10% of the actual maximum shear stress.

20.2 Nominal Strength

The strength requirement for beams in load and resistance factor design is stated as

$$
\boxed{\phi_b M_n \geq M_u} \qquad (20.10)
$$

where:

ϕ_b strength reduction factor; for flexure 0.90
M_n nominal moment strength
M_u factored service load moment.

14 The equations given in this chapter are valid for flexural members with the following kinds of cross section and loading:

1. Doubly symmetric (such as W sections) and loaded in Plane of symmetry
2. Singly symmetric (channels and angles) loaded in plane of symmetry or through the shear center parallel to the web[1].

20.3 Flexural Design

20.3.1 Failure Modes and Classification of Steel Beams

15 The strength of flexural members is limited by:

Plastic Hinge: at a particular cross section.

[1]More about shear centers in *Mechanics of Materials II*.

19 The nominal strength M_n for laterally stable compact sections according to LRFD is

$$\boxed{M_n = M_p} \tag{20.12}$$

where:
- M_p plastic moment strength $= ZF_y$
- Z plastic section modulus
- F_y specified minimum yield strength

20 Note that section properties, including Z values are tabulated in Section 16.6.

20.3.1.2 Partially Compact Section

21 If the width to thickness ratios of the compression elements exceed the λ_p values mentioned in Eq. 20.11 but do not exceed the following λ_r, the section is partially compact and we can have local buckling.

$$\boxed{\begin{array}{rlll} \text{Flange:} & \lambda_p < \frac{b_f}{2t_f} \leq \lambda_r & \lambda_p = \frac{65}{\sqrt{F_y}} & \lambda_r = \frac{141}{\sqrt{F_y - F_r}} \\ \text{Web:} & \lambda_p < \frac{h_c}{t_w} \leq \lambda_r & \lambda_p = \frac{640}{\sqrt{F_y}} & \lambda_r = \frac{970}{\sqrt{F_y}} \end{array}} \tag{20.13}$$

where, Fig. 20.6:
- F_y specified minimum yield stress in ksi
- b_f width of the flange
- t_f thickness of the flange
- h_c unsupported height of the web which is twice the distance from the neutral axis to the inside face of the compression flange less the fillet or corner radius.
- t_w thickness of the web.
- F_r residual stress = 10.0 ksi for rolled sections and 16.5 ksi for welded sections.

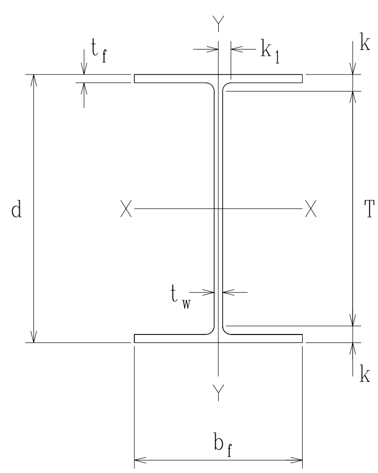

Figure 20.6: W Section

22 The nominal strength of partially compact sections according to LRFD is, Fig. 20.7