Hydraulic Engineering
Prof. Mohammad Saud Afzal
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture – 12
Laminar and turbulent flow

Welcome back to this week's lecture of hydraulic engineering. We are going to study about laminar and turbulent flows. This is the week 3 of this module and this will comprise of almost 5 lectures of half an hour each. So, proceeding to laminar and turbulent flow, its important to tell you about what these types of flows are.

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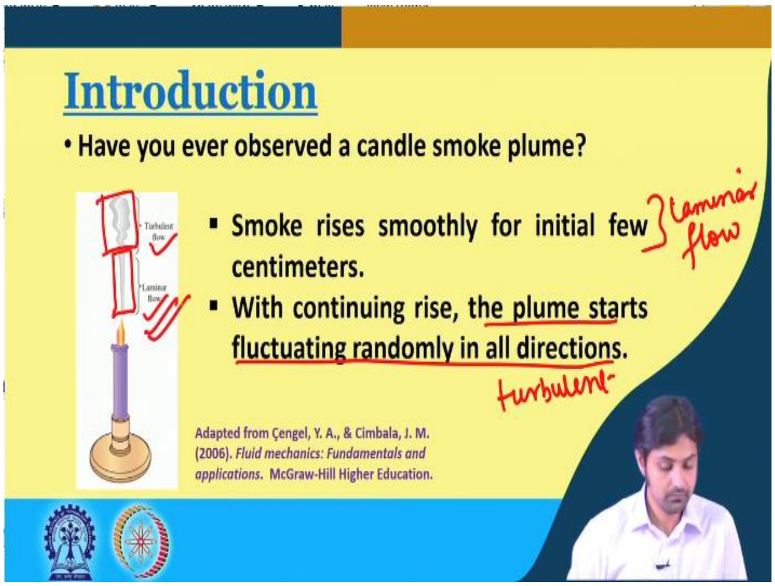

So, my question to you is, have you ever observed the candle smoke plume? If you have observed, you would note that when the smoke plume above the candle flame there will be a smooth part, you know, and there will be a rough part of the smoke. So, this part actually indicates this laminar flow and this is the one that is turbulent flow. We will go into more detail about what laminar and turbulent flow now.

As I said, what you are going to observe is that smoke actually rises smoothly for initial few centimeters. And then without, you know, further going upwards maybe just a little bit upward after going that the plume starts fluctuating randomly in all direction or, you know, in other words, the flow becomes turbulent and this is indicative of laminar flow. The figure has been adapted from Cengel's book Fluid Mechanics: Fundamentals and Application, McGraw-Hill Higher Education Publication. So, as I said, the flow regime in the first case that is the first case.

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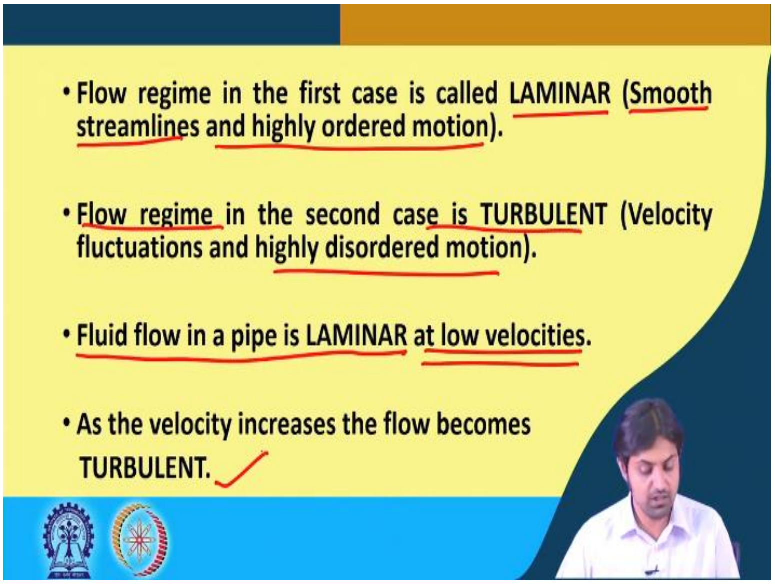

It is called the laminar flow or smooth streamlines are there and it is actually a very highly ordered motion, so, no problems so at all, very, very smooth. In the second case the flow regime is turbulent, this means, the velocity fluctuations are there and it is a highly disordered motion. So, any chaos, you know, any fluctuations in the velocity is called turbulence. And this is the flow that is associated with this is called a turbulent flow. So, what we observe is fluid flow in a pipe is laminar when at low velocities.

So, if the velocity is very low the flow in the fluid can be laminar, and as the velocity increases the flow becomes turbulent. So, higher velocities are associated with turbulent flow and lower velocities are associated with laminar flows, this is the most general thing. But it also depends on some other parameters which we will see in the upcoming slides. So, if the flow is laminar whether the flow is turbulent it does not only depend on the velocities.

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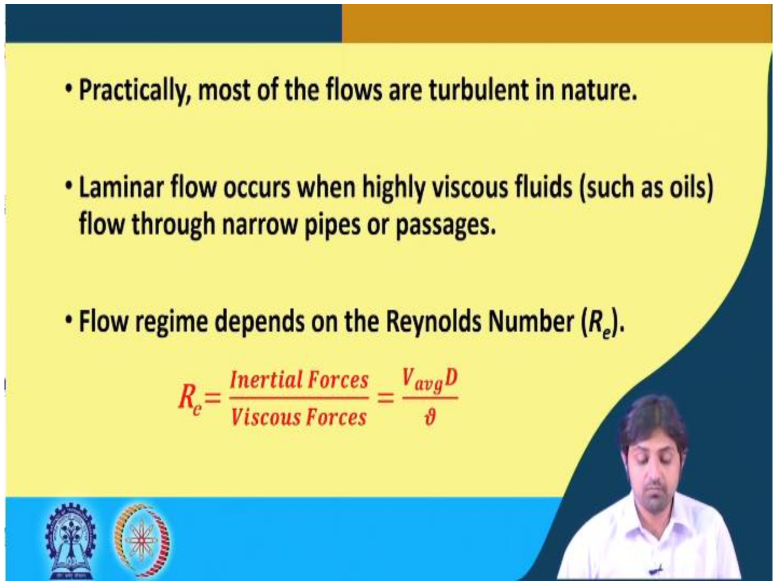

What happens in nature practically most of the flows in the nature are at turbulent, most of flow, we rarely find laminar flow. One real-life example of a laminar flow actually occurs in the our blood system. So, in the veins and in the arteries the flow can be laminar, actually it is laminar. We will tell you the reasons why later. Laminar flow can also occur when a highly viscous fluids, such as, oil flow through a narrow pipe or passages.

So, if you pay attention to what I have said just now, just before that, why the flow in the blood is laminar, you can actually understand. So, this is this could be the, you see, narrow pipes. We will see numerically also why soon, why the flow in the blood is laminar. Now the factor on which the flow regime, flow regime means whether it is laminar or turbulent depends on the Reynolds number Re. What is this Reynolds number? Reynolds number is a dimensionless number which is the ratio of the inertial forces divided by the viscous forces sorry.

So, it is the ratio of inertial forces to viscous forces or inertial forces by viscous forces. Mathematically it is given as, V average multiplied by D divided by nu, so, this has something to do with this viscosity, that is, why viscous forces, you see, and this has something to do with inertia. This Reynolds equation for Reynolds number is one of the most important thing in this course, it will you will encounter the Reynolds number more than you can imagine in this course. So, I would recommend that you remember this Reynolds number as inertial forces by viscous forces.

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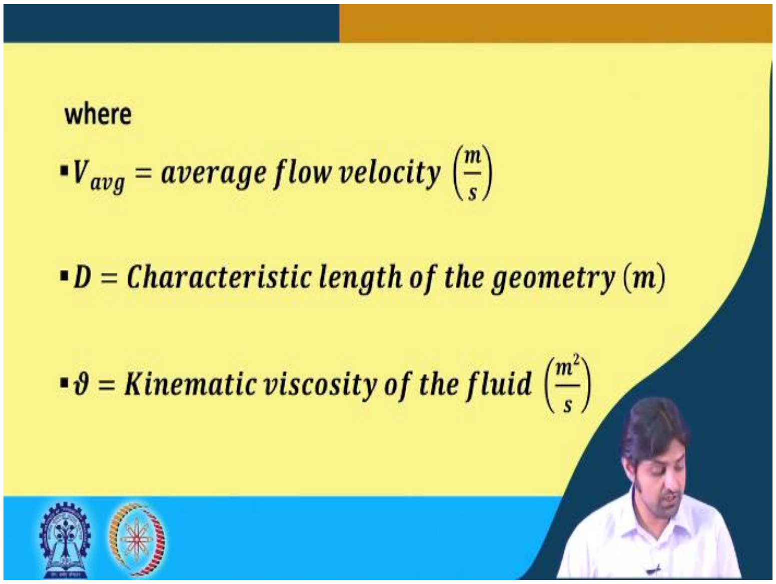

Here, V average is the average flow velocity in meters per second, D is the characteristic length of the geometry, it can be anything, it could be the diameter, it could be the length and nu is the kinematic viscosity of the fluid. This can vary from, you know, problem to problem or geometry to geometry. For example, if you consider sand grain it has it can be nothing other than the sand diameter, if you consider a pipe it cannot be anything else, other than, the pipe diameter, for example. So, Reynolds number is very important, you have to note that again. So, velocity into characteristic length by viscosity.

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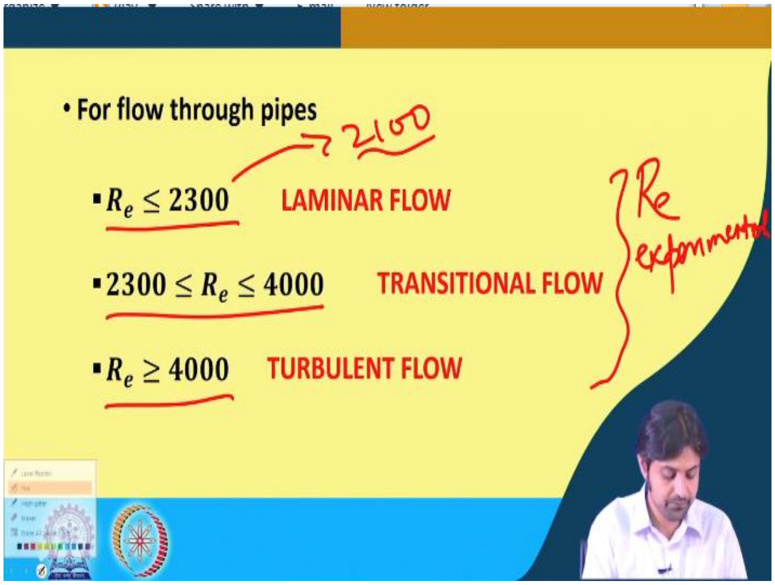

For example, for flow through pipes, if the Reynolds number is less than 2300, we consider the flow as laminar flow, important to remember. So, if you are given, the pipe diameter, the flow velocity, you can easily tell if the flow is laminar or turbulent. What you can do is, you can calculate Reynolds number as VD by nu, this is known, this is told and D is also told. So, you can calculate Reynolds number. For Reynolds number between 2300 and 4000 the flow is transitional.

So, there is one other category of flow apart from laminar and turbulent called transitional flow, which means that the laminar regime has just ended and the fully turbulent regime has not yet started. So, some of the flow could be laminar, sorry, the flow could have still some laminar properties and some turbulent. But when the Reynolds number goes over and above 4000 for pipe flow this becomes, the flow becomes fully turbulent, this is you know a guideline.

So, remembering all these three values is not a bad idea. If you also, I mean, some books will say 2100 as well, but the idea is it should be around 2000 laminar flow. So, around 2000, 22-2300 is the value recommended by many books. All these velocities, I mean, the all these velocity measurements and Reynolds number measurement is experimental in nature.

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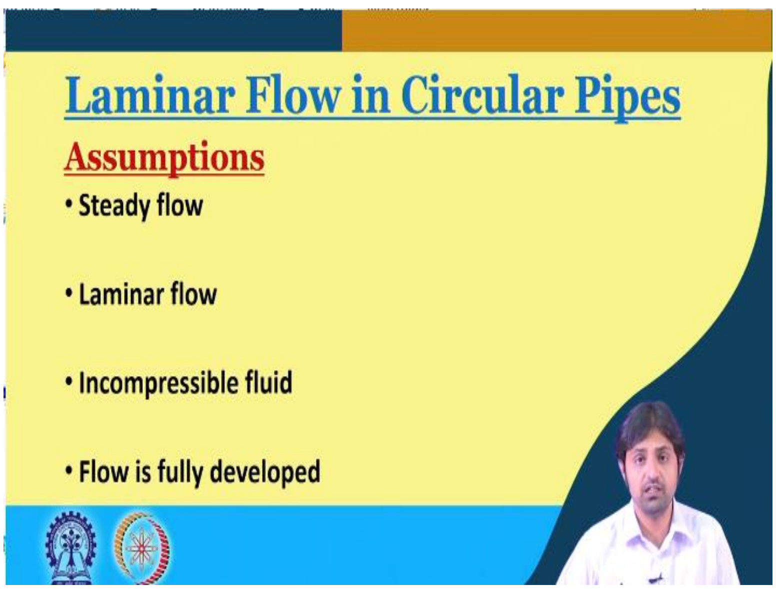

So, now laminar flow in circular pipes, so, after dealing telling you the basics of how to define the, you know, how to define and find what laminar and turbulent flow is, we are going to see some of the properties of laminar flow in circular pipes. So, for deriving anything there are certain assumptions first that we have to take. We have to assume steady flow, what does this assumption of steady flow mean, that the situation or the condition is not dependent on time.

Second assumption is, it is laminar flow, this means the Reynolds number is fairly less than 2300, as we have seen in the last slide. And the flow is incompressible which means density is constant, the density does not changes either in space or with time. And we also have to assume the flow is fully developed, that means there is no, you know, intermittent phenomenon that is happening, a flow has happened over a long period of time. And then we are going to calculate the properties.

Because when the flow start the situation it is actually first thing that will happen is it being non-steady, you know, it will change the function. After flow is fully developed, you know, the flow has happened over a long period of time and then the flow becomes fully developed it has occupied the entire pipe, for example, so, it is fully developed. Fully developed has something to do with the boundary layer phenomenon that is why I have not mentioned it before but we will see it in the next week.

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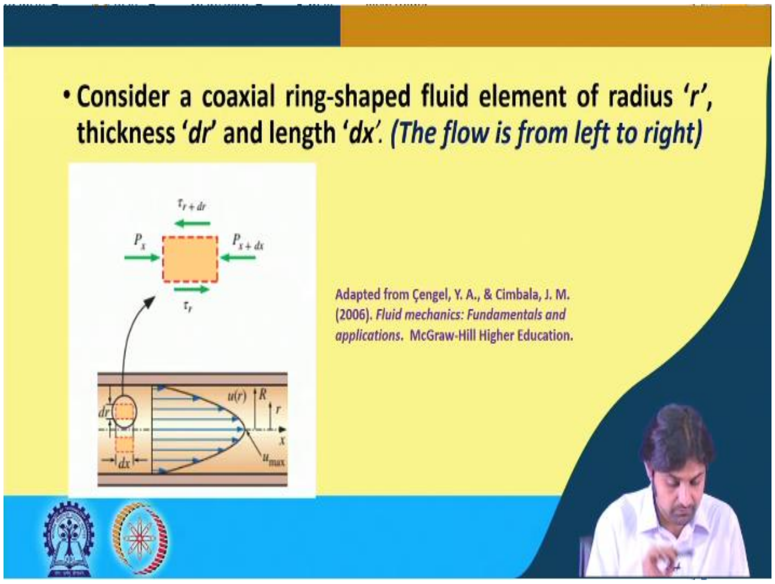

Now, after going through the assumptions, we will consider a coaxial ring shaped fluid element of radius 'r' whose thickness is 'dr' and length is 'dx' and the flow is from left to right. As I said, the best way to describe a problem is to draw it. So, what we have assumed is, we have assumed a coaxial ring shaped fluid element of radius ‘r’. So, this is the flow, you know, this is the profile and we have assumed this fluid element, which having thickness ‘dr’ and length ‘dx’ and the flow is happening from left to right in this direction.

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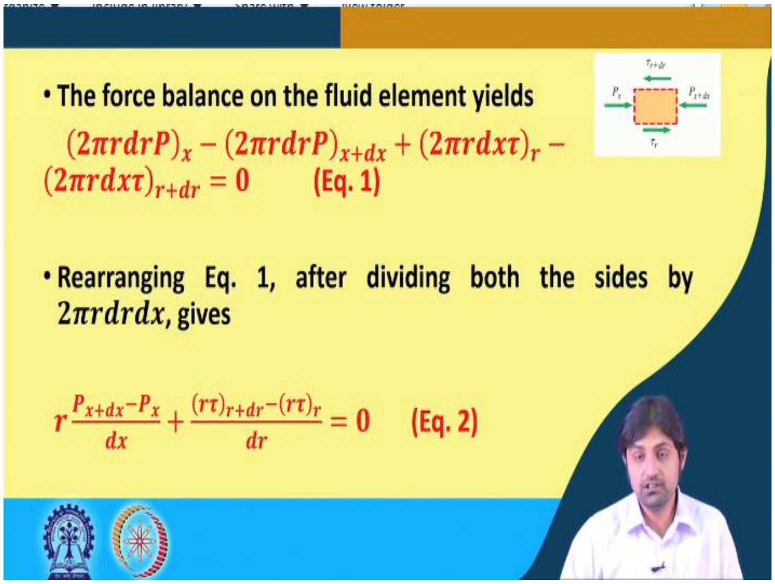

So, we have kept this figure for the forces in the right hand side. So, if you see, there will be pressure forces acting P x from the left, P x + P pressure force are at x + dx from the right and then there will be shear forces acting here, in this direction and there will be shear forces acting at Tr + dr, you know, and if we apply the force balance, so,

$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx \tau)_r - (2\pi r dx \tau)_{r+dr} = 0$$

Now, this one, force acting in this direction which is positive in sign and there is another force acting in the negative direction, so, that is what we have done. So, again this is a shear force, so, we have multiplied this tau here, the one here, with 2 pi r dx, because this is the thickness over which it is, I mean, so, this is the length over which it is acting. So, this was if you see, this was dx and this was dr. So, multiplication of dr was done for pressure at, I mean, for this force and this force, however, multiplication with x will be done for the tau shear stress and the shear stress here.

So, this was positive sign, 2 pi r x dx in the area and the shear stress is tau at r minus, because this is the minus 1, because it is in the negative x direction, same area 2 pi r dx into tau at r + dr.

So, this denotes where this force is, it is at x, this is at x + dx, this is at r and this is at r + dr location. So, now if we rearrange this equation here, equation 1 and we divide both sides by 2 pi r dr into dx.

So, this equation will come out to be, r P x + dx - P x by dx, because 2 pi r dr will be cancelled from here and here. So, this is the first term and the second term will happen. So, here, 2 pi r dx will be cancelled out and the dr will be remaining. So, it will be this term and both will be sum will be equal to 0 very simple to obtain, just considering this and this. So, this is another equation that we call equation number 2.

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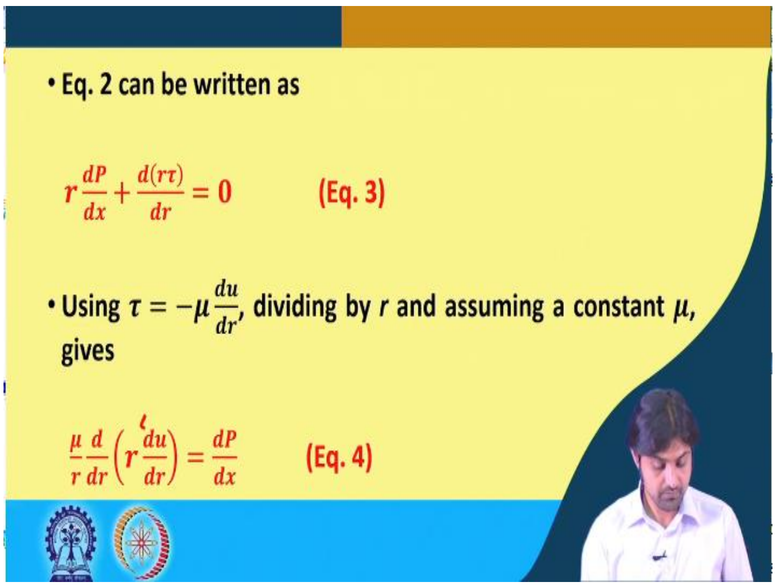

Now, equation number 2 can be written as simply you see, so this is pressure at x + dx - P x by dx, so it can be written as, dP dx and this can be written as, d r into tau dr. So, this is the differential form of the equation. Now, if we use the standard tau is equal to minus mu du dr, why do we do this? Actually this is an assumption for laminar flow. So, if we have a laminar flow we can assume shear stress as a function of minus, you know, as a function of du dr or in other terms tau is equal to minus mu du dr.

So, minus mu du by dr, this is laminar flow. So, if we use this equation here, then what we get? We can get, mu by r d dr of r du dr is equal to dP dx because this minus will make it come on this side and this dP dx can be on the left side, right side.

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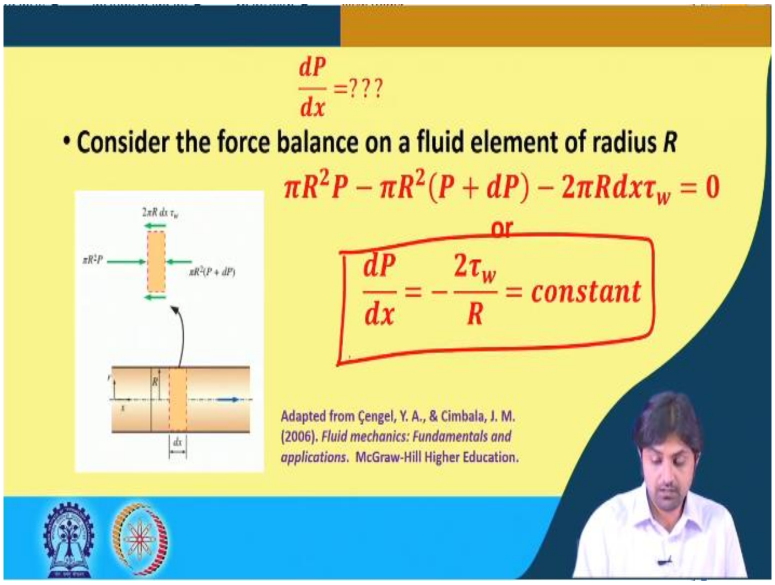

So, now what is dP dx? So, to find dP dx, we have to consider the force balance on fluid element of radius R. So, we consider a fluid element of radius R, it is quite simple, their pressure here is P. So, it becomes pi R square P, this force and this force is pi R square P + dP and the same thing is there is the shear stress acting here. So, this is the tau w. So, almost the same type of equation is there, pi R square P - pi R square P + dP force balance - 2 pi Rdx tau w.

So, this is the wall. So, what we get is, if you see, the equation that we get here is,

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

which is a constant. So, if we integrate this equation, equation 4, so, this is the equation 4, after obtaining this result here.

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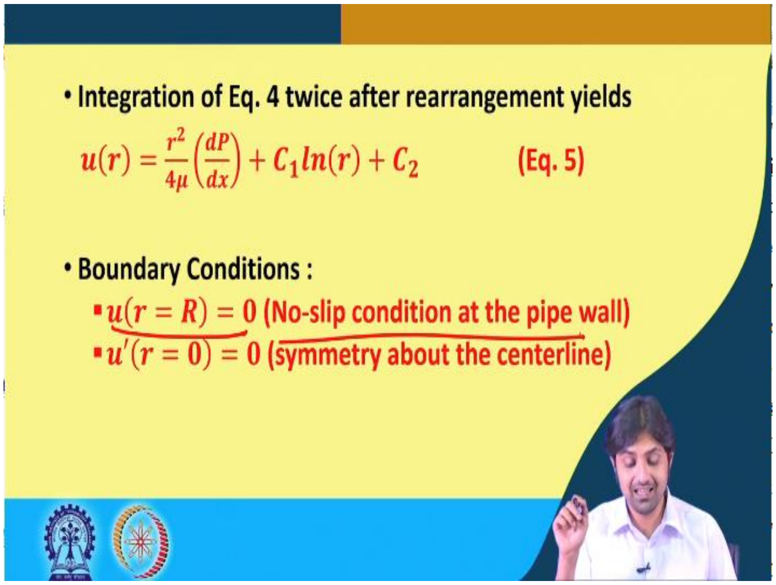

What we are going to get if we integrate this? We will get, u as a function of r is

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 ln(r) + C_2$$

, this is an important equation. So, now we will talk about some boundary conditions. Boundary conditions are something that we used to determine the constants of these equations. So, one of the boundary condition is that we had actually used in the derivation of dP dx that, you know, u is equal to 0 at when radius is equal to capital R. This is called no slip condition at pipe wall because pipe wall is at rest.

So, the fluid element adjacent to the wall will also be at rest. So, u at r is equal to small r is equal to capital R is 0. Another thing we will have the uPrime at R is equal to 0 or at the center line it is going to be 0 that is symmetry about the center line.

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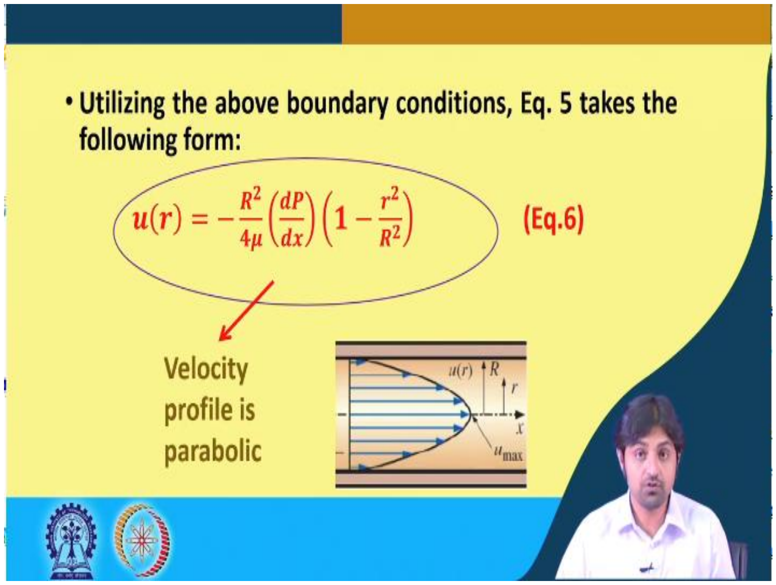

So, if we utilize the above boundary conditions, equation 5 will take the following form. So, C 1 and C 2 will be eliminated and we will have u as a function of r will be given as

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right)$$

. And this is velocity profile which is parabolic in nature, this is important to note, you see, this part here is parabolic.

So, this now you can recollect and see why we started with this particular profile because in reality in derivation you will find this profile which is parabolic in nature. So, u as a function of r if you find out if you put r is equal to R at boundary just check u R, so is going to be 0 this is satisfied. But actually we use this assumption to derive, so, we should not say that this is actually a check of the equation because of course it is going to be that because that was the boundary condition.

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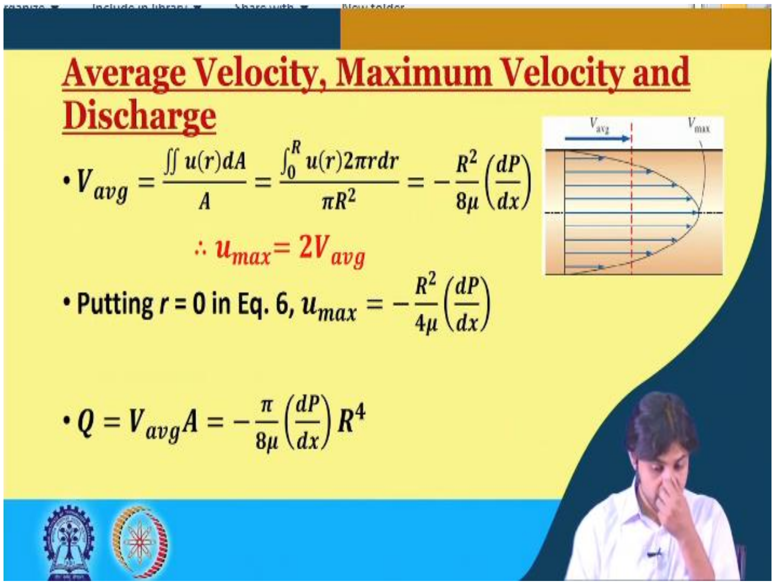

Now, what is being what will be the average velocity, maximum velocity and the discharge in this type of flow. So, the average velocity will be integration of velocity with a small area divided by entire area, this is the definition of average velocity. And if you put, u r as what we have obtained from the last slide and if you assume that small element it is the area will be 2 pi r dr, correct.

So, if you put u r is equal to this and area is 2 pi r dr, then you are going to obtain the average velocity as minus R square by 8 mu multiplied by dP dx, this is again an important. So, these are some of the top, I mean, things that you get in gate as well gate. So, now you have obtained the average velocity. So, if you see, the velocity profile, parabolic profile you can easily actually see the maximum velocity, sorry, the maximum velocity occurs at r is equal to 0.

So, if we put r is equal to 0 in the equation from the last slide, that is, equation 6 u max will be

$$u_{max} = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)$$

. And it is very, if you see the relation, u max is equal to 2 times u average, if you divide this, divide this by divide this you will get 1 by 2. So, you see, so, this is the V average here, and this is the u or V whatever you want to say, or u max is equal to 2 V average and the simply the total discharge is going to be V average into a, that is, minus pi. So, V average we already know, and you multiply the area. So, you will get, so, it is R square already, and then what do you multiply with pi R square, and this is the Q that you are going to get.

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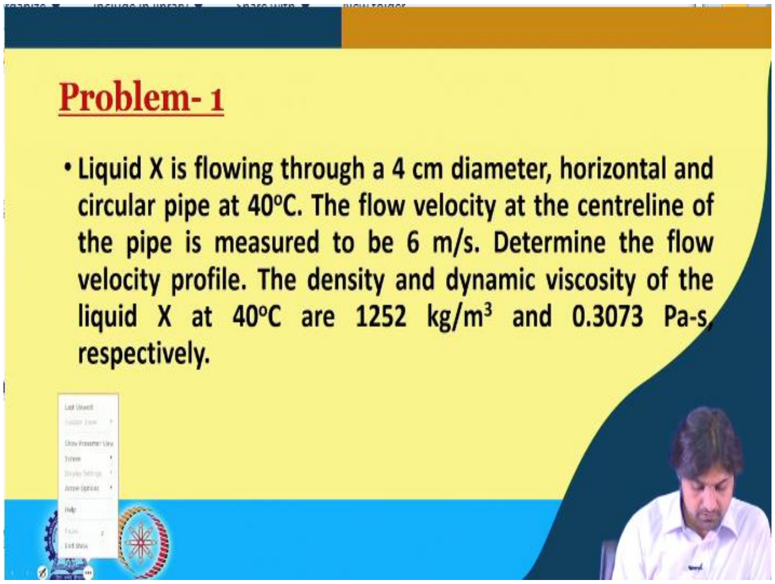

So, now we are going to see the first problem from the laminar flow. So, the problem is, there is a liquid X which is flowing through a 4 centimeter diameter, horizontal and a circular pipe at 40 degree centigrade. This is what we have done before. The flow velocity at the centerline of the pipe is measured to be 6 meter per second. So, basically what this says we have measured the centerline velocity of the pipe.

We have to determine the flow velocity profile. The density and the dynamic viscosity of the liquid x at 40 degree centigrade are, so, they are telling us the density is this one 1252 kilogram per meter cube and dynamic viscosity is point 3073 Pascal second. So, they have told a certain piece of information and we have to solve this problem. So, how do we do that. Now I what I am going to do is, I am going to use the white screen for that and the best way is first we have to write what are the things that are given.

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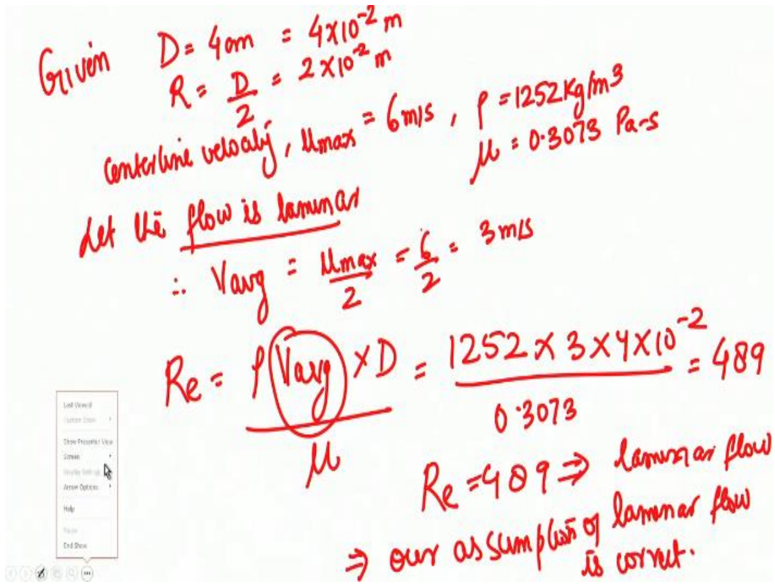

So, we have to write the things that are given. So, we are given the diameter is equal to 4 centimeter, which is 4 into 10 to the power minus 2 meters. Therefore, the radius is going to be D by 2 or simply, 2 into 10 to the power minus 2 meter. We are also given the centerline velocity which is, we have seen in our derivation as, u max, that is, 6 meters per second. We are also told that density is 1252 kilogram per meter cube and also the kinematic viscosity in SI unit is point 3073 Pascal second.

So, what was the first thing that we did? We wrote down everything that is given. This is the very good practice. So, this is for an assumption that the flow is laminar, so, we say let the flow is laminar. Therefore, V average is going to be u max / 2 and, that is, 6 / 2 is equal to 3 meters per second. So, now we have assumed that the flow is laminar, we have to check also. Therefore, first, we have to check the Reynolds number we say, Reynolds number is rho V average.

This is one term multiplied by D by mu. Because in the definition that we saw, it was nu and nu is mu by rho, so rho goes up, so it is 1252 into 3 into 4 into 10 to the power minus 2 divided by point 3073 and that comes almost 489. So, Reynolds number is without dimension that is very, 489 which implies that this is laminar flow and therefore our assumption of laminar flow is correct. So, we will go to another page. So, we will go to white screen again.

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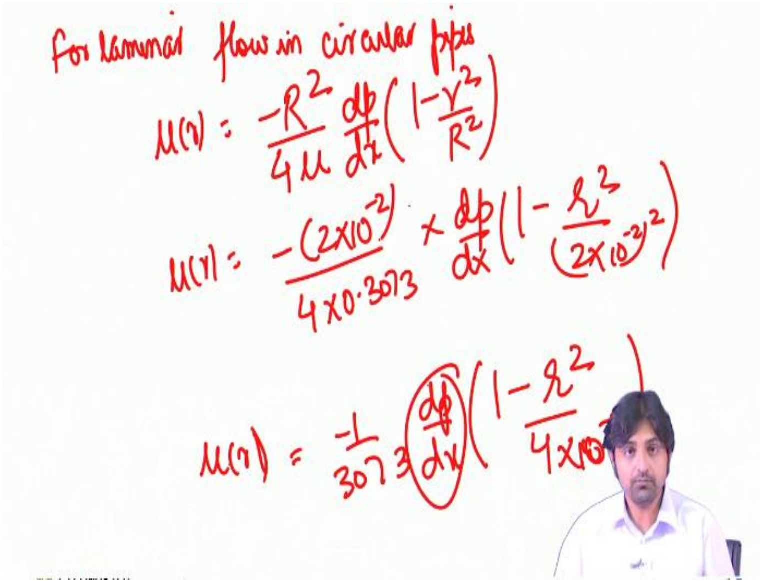

So, we say for laminar flow in circular pipes u of r is written as very, minus R square by 4 mu dP dx, this was the formula that we derived already and we know many values. So, we can simply write, minus R, we already know, 2 into 10 to the power minus 2 divided by 4 into 0.3073, dP dx we do not know into 1 - r square, capital R we know, 2 into 10 to the power minus 2 whole square. So, this is the profile that we are able to find. If you want to simplify it further, we can simply write, u of r as minus 1 by 3073 dP dx into 1 - r square 4 into 10 to the power minus 4.

So, we have derived u as a function of r. dP dx is something that we do not know, I mean, or if we know this then our problem will be solved. So, I will just go to the screen on white screen and this is what was asked, to determine the flow velocity profile. So, now I think we will start with another problem in our next lecture. So, I will just showing you the problem I am not going to repeat it this question because we will start our next lecture with this particular problem. So thank you for watching and we resume in the upcoming lecture, thank you so much.