(a) Linear methods of setting out curves

The following methods of setting out simple circular curve are linear as measurement is doen using chain/tape/distance/EDM:

1. By ordinates from the long chord
2. By successive bisection of arcs
3. By offsets from the tangents
4. By offsets from chords produced

1. By ordinates from the long chord:

In this method, the perpendicular offsets are erected from the long chord to establish points along the curve, as shown in Figure 2.5.

If T_1T_2 is the length of the long chord (L), ED = O_0 which is the offset at mid-point (E) of the long chord (*the versine*), and PQ = O_x which is the offset at distance x from E. Draw a line QQ_1 parallel to T_1T_2 which meets OD at Q_1, and line OQ_1 which cuts T_1T_2 at point E.

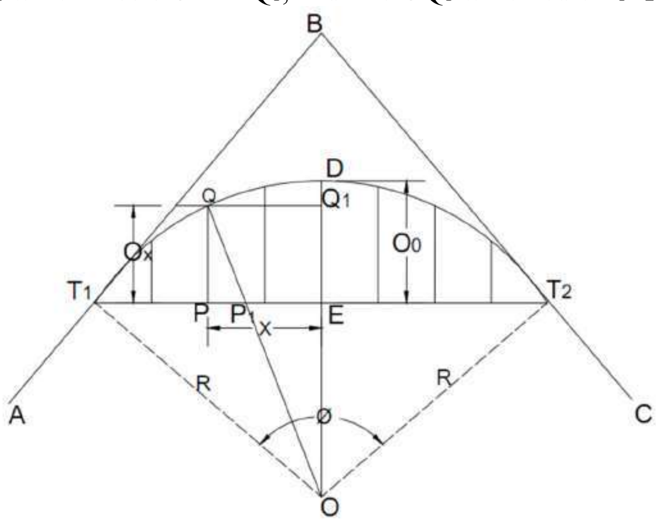

Figure 2.5 Setting out the curve by ordinates from the long chord

$OQ_1 = OE + EQ_1$

$= (OD - DE) + EQ_1$

$= (R - O_0) + O_x$

From ΔOQQ_1

$OQ^2 = QQ_1^2 + OQ_1^2$

But OQ = R, and $QQ_1 = x$

$R^2 = x^2 + \{(R - O_0) + O_x\}^2$

or $\quad (R - O_0) + O_x = \sqrt{R^2 - x^2}$

$$\text{Hence } O_x = \sqrt{R^2 - x^2} - (R - O_0) \qquad (2.9)$$

$OE = \sqrt{(OT_1^2 - T_1E^2)}$

$= \sqrt{[R^2 - (L/2)^2]}$

Where $O_0 = ED = OD - OE$

But OE = $\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$

So $\quad O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$ (2.10)

In relationship 2.9, the value of O_0 may be replaced as-

$$\text{Hence } O_x = \sqrt{R^2 - x^2} - [\, R - \{\, R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2} \,\}] \quad (2.11)$$

The curve is set out as below:

(i) Divide the long chord into an even number of equal parts, if possible.
(ii) Calculate the offsets using equation (2.11) at each of the points of division.
(iii) Set out the offset at respective points on the curve.
(iv) Since, the curve is symmetrical about the middle-ordinate, therefore the offsets for the right-half of curve will be same as those for the left–half curve.
(v) The method is suitable for setting out short curves e.g., curves for street bends.

2. By successive bisections of arcs:

It is also known as *Versine method*. In Figure 2.6, a curve T_1DT_2 is to be established on the ground by this method. The steps involved are-

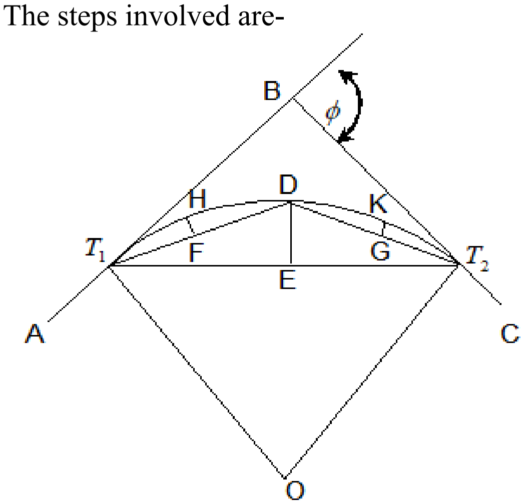

Figure 2.6 Setting out the curve by successive bisection of arcs

(i) Join T_1T_2 and bisect it at E. Set out the offset ED (which is equal to the versine $R\left(1-\cos\frac{\phi}{2}\right)$, thus a point D on the curve may be fixed.

(ii) Join T_1D and DT_2 and bisect them at F and G, respectively. Then set out the offsets FH and KG at F and G, respectively, in the same manner, each equal to $R\left(1-\cos\frac{\phi}{4}\right)$. Thus, two more points H and K are fixed up on the curve.

(iii) Now, each of the offsets can be set out at mid points of the four chords T_1H HD, DK and KT_2 which is equal to $R\left(1-\cos\frac{\phi}{8}\right)$.

(iv) By repeating this process, several points may be set out on the curve, as per the need.

(v) This method is suitable where the ground distance outside the curve is not favorable for measurements by tape.

3. By offsets from the tangents: The offsets may be either redial or perpendicular to the tangents.

(a) By radial offsets

In Figure 2.7, if $O_x = PP_1$ which is the radial offset at P from O at a distance of x from T_1 along the tangent AB, then-

$PP_1 = OP - OP_1$ where $OP = \sqrt{R^2 + x^2}$ *and* $OP_1 = R$

$$O_x = \sqrt{R^2 + x^2} - R \qquad \text{(Exact)} \qquad (2.12)$$

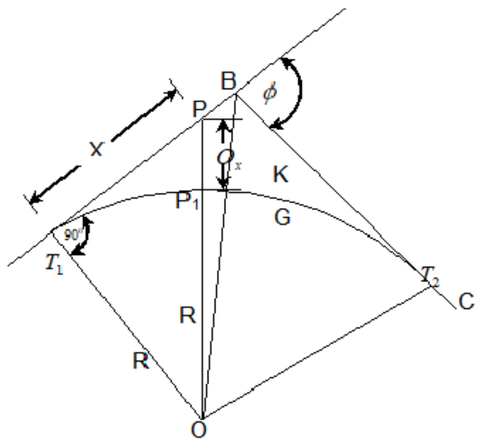

Figure 2.7 Setting out the curve by radial offsets from the tangents

When the radius of curve is large, the offsets may be calculated by the approximate formula as derived below. Using the property of a circle, we can write;

$$PT_1^2 = PP_1 \times (2R + PP_1)$$

$$x^2 = O_x (2R + O_x) = 2RO_x + O_x^2$$

Since O_x^2 is very small as compared to 2R, it may be neglected. Hence,

$$x^2 = 2R\, O_x$$

or $$O_x = \frac{x^2}{2R} \qquad \text{(approximate)} \qquad (2.13)$$

(*b*) By offsets perpendicular to the tangents

In Figure 2.8, $O_x = PP_1$ which is the perpendicular offset at P at a distance of x from T_1 along the tangent AB. Draw P_1P_2 line parallel to BT_1

$P_1P_2 = PT_1 = x$, and $T_1P_2 = PP_1 = O_x$

Now $T_1P_2 = OT_1 - OP_2$

Where $OT_1 = R$, and $OP_2 = \sqrt{R^2 - x^2}$

So $$O_x = R - \sqrt{R^2 - x^2} \qquad \text{(exact)} \qquad (2.14)$$

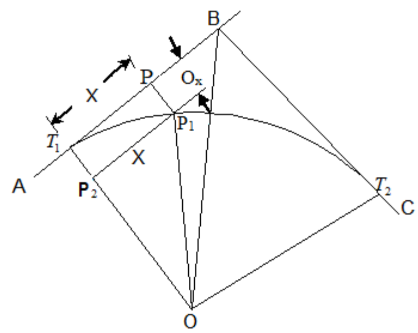

Figure 2.8 Setting out the curve by offsets perpendicular to the tangents

Approximately, the formula may be obtained similarly as equation 2.13;

$$O_x = \frac{x^2}{2R} \qquad \text{(approximate)} \qquad (2.15)$$

Procedure of setting out the curve:

(i) Locate the tangent points T_1 and T_2.
(ii) Measure equal distances, say 15 or 30 m along the tangent from T_1.
(iii) Set out the offsets calculated by any of the above methods at each distance (say x), thus obtaining the required points on the curve.
(iv) Continue the process until the apex of the curve is reached.
(v) Set out the other half of the curve from the second tangent; being symmetrical in nature.

This method is found to be suitable for setting out sharp curves where the ground outside the curve is favorable for measuring the distance.

5. By offsets from chords produced

In Figure 2.9, if AB is the first tangent, T_1 is the first tangent point, E, F, G etc., are the successive points on the curve. Draw arc EE_1, so $T_1E = T_1E_1 = C_1$ which is the first chords. Similarly, EF, FG, etc., are successive chords of length C_2, C_3 etc., each being equal to the full chord length. $\angle BT_1E = \alpha$ in radians (angle between tangent BT_1 and the first chord T_1E). $E_1E = O_1$ which is the offset from the tangent BT_1, and $E_2F = O_2$ which is the offset from the chord T_1E produced. Produce T_1E to E_2 such that $EE_2 = C_2$. Draw the tangent DEF_1 at E meeting the first tangent at D and E_2F at F_1, then-

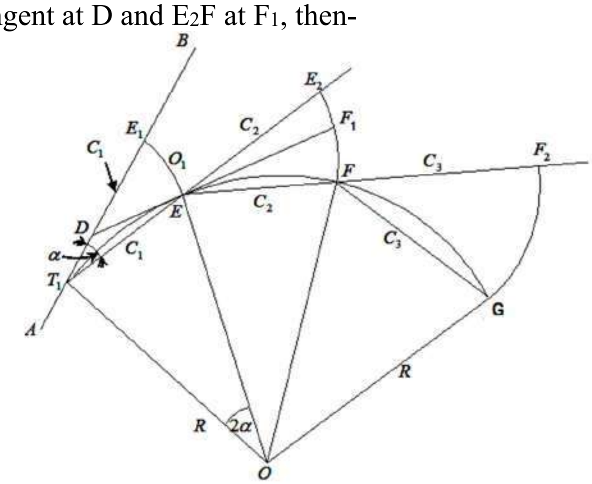

Figure 2.9 Setting out the curve by offsets from the chord produced

$\angle BT_1E = \alpha$

$\angle T_1OE = 2\alpha$ (The angle subtended by any chord at the centre is twice the angle between the chord and the tangent

$$\frac{arc T_1E}{Radius\ OT_1} = 2\alpha$$

But arc T_1E may be taken as approximately equal to chord $T_1E = C_1$

so, $\frac{C_1}{R} = 2\alpha$ or $\alpha = \frac{C_1}{2R}$ (2.16)

also $\frac{arc\ E_1E}{T_1E} = \alpha$

But arc E_1E is approximately equal to chord $E_1E = O_1$, and $T_1E = C_1$, so

$O_1 = C_1 \times \alpha$

Putting there the value of α as calculate above.

$$O_1 = C_1 \times \frac{C_1}{2R} = \frac{C_1^2}{2R} \quad (2.17)$$

Now O_2 = offset $E_2F = E_2F_1 + F_1F$

To find out F_2F_1, consider the two triangles T_1EE_1 and EF_1E_2

$\angle E_2EF_1 = \angle DET_1$ (vertically opposite angles)

$\angle DET_1 = \angle DT_1E$, since $DT_1 = DE$, both being tangents to the circle

$\angle E_1FF_1 = \angle DET_1 = \angle DT_1E$

Both the triangles being nearly isosceles, may be considered as approximately similar. So we may write-

$$\frac{E_2F_1}{EE_2} = \frac{E_1E}{T_1E_1}$$

$$\frac{E_2F_1}{C_2} = \frac{O_1}{C_1}$$

or $E_2F_1 = \frac{C_2 \times O_1}{C_1}$

or $= \frac{C_2}{C_1} \times \frac{C_1^2}{2R} = \frac{C_1C_2}{2R}$

F_1F being the offset from the tangent at E, is equal to

$$\frac{EF_2}{2R} = \frac{C_2^2}{2R}$$

O_2 = offset $E_2F = E_2F_1 + F_1F$

$$O_2 = \frac{C_1C_2}{2R} + \frac{C_2^2}{2R}$$

$$= C_2\left(C_1 + C_2\right) / 2R \quad (2.18)$$

Similarly, the third offset, $O_3 = \frac{C_3\left(C_2 + C_3\right)}{2R}$ (2.19)

In the same way, remaining offset O_4, O_5, etc. may be computed using the general relationship.

$$O_n = \frac{C_n(C_{n-1} + C_n)}{2R} \quad (2.20)$$

Since $C_2 = C_3 = C_1$ ……………… etc., so equations 2.18 and 2.19 may be written as;

$O_2 = \frac{C_2^2}{R}$ and

$$O_3 = \frac{C_2^2}{R} \quad (2.21)$$

It is to be noted that first and last offsets may have different length (due to the chainages of their chord lengths), while all intermediate offsets will have equal length.

Procedure of setting out the curve:

(i) Locate the tangent points (T_1 and T_2) and find out their chainages. From these chainages, calculate the lengths of first and last sub-chords and find out the offsets by using above equations.

(ii) Mark a point E_1 along the first tangent T_1B such that T_1E_1 equals the length of the first sub-chord.

(iii) With the zero end of the tape at T_1, swing an arc E_1E equal to radius T_1E_1, and mark point E such that $E_1E = O_1$, thus fixing the first point E on the curve.

(iv) Line T_1E is produced and E_2 equal to the second sub-chord is marked, and an arc from E_2 is drawn such that $EE_2 = EF$ to locate the second point F on the curve.

(v) Continue this process until the end of the curve is reached.

(vi) The last point fixed in this way should coincide with the previously located point T_2. If there is small closing error, all points on the curve are moved sideways by an amount proportional to the square of their distances from the tangent point T_1, but if the error is large, the entire process is repeated.

This method is commonly used for setting out road curves.

(b) Angular methods of setting out curves

There are two methods of setting out simple circular curves by angular methods:

1. Rankine's method of tangential angles
2. Two theodolites method

1. Rankine's method of tangential or deflection angles:

In Rankine's method, the curve is set out by the tangential or deflection angles using a theodolite and a tape. The deflection angles are calculated to set out the curve (Figure 2.10).

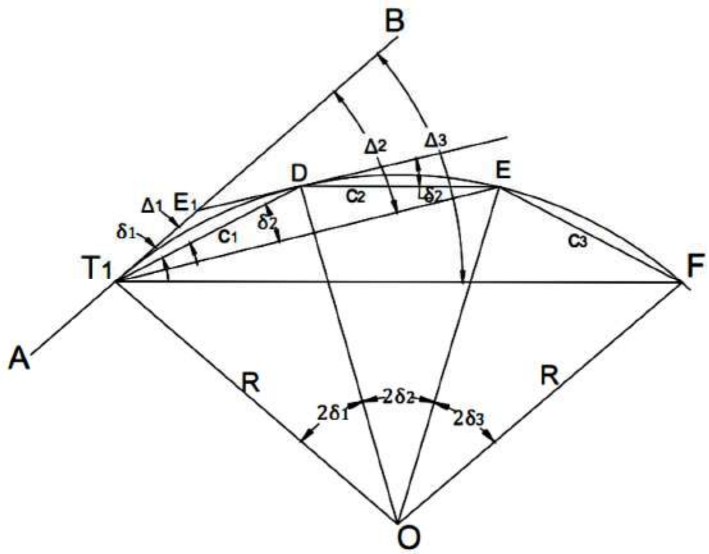

Figure 2.10 Curve setting by Rankine's method

If T_1 and T_2 are the tangent points and AB the first tangent to the curve, D, E, F etc., are the successive points on the curve, ϕ is the deflection angle of the curve, R is the radius of the curve, C_1, C_2, C_3, etc., are length of the chords T_1D, DE, EF etc.., $\delta_1, \delta_2, \delta_3$ *etc.* are the tangential angles which each of the chords T_1D, DE, EF, etc., makes respectively with the tangents at T_1, D, E, etc., Δ_1, Δ_2, Δ_3 etc., are the total tangential or deflection angles which the chords T_1D, DE, EF, etc. make with the first tangent AB, then-

The chord T_1D can be taken as equal to arc $T_1D = C_1$

$$\angle BT_1D = \delta_1 = \frac{1}{2}\angle T_1OD = 2\delta_1$$

$$\frac{arc\ T_1D}{radius\ OT_1} = \angle T_1OD\ in\ radians$$

or $\quad \frac{C_1}{R} = 2\delta_1$ radians

or $\quad \delta_1 = \frac{C_1}{2R} radians$

$$= \frac{C_1}{2R} x \frac{180}{\pi}\ degrees$$

$$\delta_1 = \frac{C_1}{2R} \times \frac{180}{\pi} \times 60\ minutes$$

$$\delta_1 = 1718.9\ C_1 / R\ minutes \qquad (2.22)$$

$\delta_2 = 1718.9\frac{C_2}{R}$, $\delta_3 = 1718.9\frac{C_3}{R}$, and so on

So we can write a general relation as:

$$\delta_n = 1718.9\frac{C_n}{R}\ minutes \qquad (2.23)$$

Since each of the chord length C_2, C_3, C_4..........C_{n-1} is equal to the length of the full chord, so $\delta_2 = \delta_3 = \delta_4 .... = \delta_{n-1}$.

The total tangential angle (Δ_1) for the first chord (T_1D)

$= \angle BT_1D = \delta_1$

So, $\Delta_1 = \delta_1$

The total tangential angle (Δ_2) for the second chord (DE) = $\angle BT_1E$

But $\angle BT_1E = \angle BT_1D + \angle DT_1E$

Since, the angle between the tangent and a chord equals the angle which the chord subtends in the opposite segment, so $\angle DT_1E$ is the angle subtended by the chord DE in the opposite segment, therefore, it is equal to the tangential angle (δ_2) between the tangent at D and the chord DE.

$$\Delta_2 = \delta_1 + \delta_2 = \Delta + \delta_2$$

Similarly, $\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$

A general relationship would be as follows:

$$\Delta_n = \delta_1 + \delta_2 + \delta_3 .... + \delta n$$

$$\Delta_n = \Delta_{n-1} + \delta_n \qquad (2.24)$$

Apply check: The total deflection angle $BT_1T_2 = \Delta_n = \frac{\phi}{2}$

If the degree of the curve (D) is known, the deflection angle for a 30 m chord is equal to D/2 degrees, and that for the sub-chord of length C_1, it would be;

$$\delta_1 = \frac{C_1}{30} \times \frac{D}{2} \text{ degrees}$$

$$\delta_1 = \frac{C_1 \times D}{60}$$

$$\delta_2 = \frac{C_2 \times D}{60} \text{ and so on.}$$

$$\delta_n = \frac{C_n \times D}{60} \qquad (2.25)$$

Procedure of setting out the curve:

(i) Locate the tangent points T_1 and T_2, and find out their chainages. From these chainages, calculate the lengths of first and last sub-chords and the total deflection angles for all points on the curve.

(ii) Set up the theodolite at the first tangent point T_1.

(iii) Set the initial horizontal circle reading to zero and direct the telescope to the intersection point and bisect it.

(iv) Set the first deflection angle Δ_1 in theodolite, and direct the telescope along T_1D. Along this line, measure T_1D equal in length to the first sub-chord, thus fixing the first point D on the curve.

(v) Now set the second deflection angle Δ_2 in theodolite, and direct the line of sight along T_1E. Hold the zero end of the tape at D and swing the other end until the tape is bisected by the line of sight, thus fixing the second point E on the curve.

(vi) Continue the process until the end of the curve is reached.

(vii) The end point thus located must coincide with the previously located point (T_2). If not, the distance between them is the closing error. If it is within the permissible limit, only the last few pegs may be adjusted; otherwise the curve should be set out again.

Note: In the case of a left-handed curve, each of the value Δ_1, Δ_2, Δ_3 etc., should be subtracted from 360^0 to obtain the required value to which the reading in theodolite is to be set i.e., the *vernier should be set to (360^0-Δ_1), (360^0-Δ_2), (360^0-Δ_3), etc., to obtain the 1st, 2nd, 3rd etc., points on the curve.*

The method is highly accurate, and is most commonly used for railways and other important curves.

2. Two theodolite method

This method is very useful in the absence of distance measurement by tape, and also when the ground is not favorable for accurate distance measurement. It is a simple and accurate method but essentially requires two theodolites to set the curve, so it is not as popular method as the method of deflection angles. In this method, the popular property of a circle "*that the angle between the tangent and the chord equals the angle which that chord subtends in the opposite segment*" is used.

In Figure 2.11, if D, E, etc., are the point on the curve to be established, then-

The angle Δ_1 between the tangent T_1B and the chord T_1D i.e., $\angle BT_1D = \Delta_1 = \angle T_1T_2D$. Similarly, $\angle BT_1E = \Delta_2 = \angle T_1T_2E$, etc.

The deflection angles Δ_1, Δ_2, etc., are calculated to establish the curve using deflection angle.

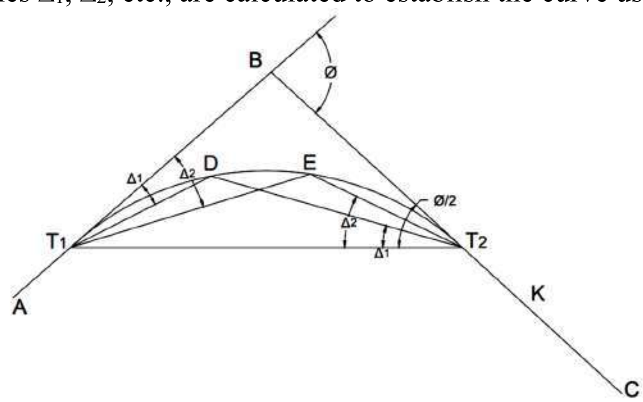

Figure 2.11 Curve setting by two theodolite method

Procedure of setting out the curve:

(i) Set up two theodolites, one at T_1 and the other at T_2.

(ii) Set horizontal angle reading of the theodolite at T_1 to zero along T_1B, and similarly the theodolite at T_2 to zero along T_2T_1.

(iii) Set both the theodolite at T_1 and T_2 to read the first deflection angle Δ_1. Now the line of sight of theodolite at T_1 would be along T_1D and that of the theodolite at T_2 along T_2D. The point of intersection of these line of sights is the required point D on the curve. Establish point D on the ground with the help of ranging rods.

(iv) Now set both the theodolites to second deflection angle Δ_2, towards T_1E and T_2E respectively, and proceed as before to establish the second point E on the curve.

(v) Repeat the process until the other points on the curve are set out.

Note: If point T_1 is not be visible from the point T_2, in such a case direct the telescope of the instrument at T_2 towards B with reading set to zero. Now set the reading to read an angle of $\left(360^0 - \frac{\phi}{2}\right)$, directing the telescope along T_2T_1. For the first point D on the curve, set the reading to read $\left(360^0 - \frac{\phi}{2}\right) + \Delta_1$. Similarly for the second point E, set the reading to read $\left(360^0 - \frac{\phi}{2}\right) + \Delta_2$ and so on.

2.4 Compound Curves

A compound curve consists of two different radii, as shown in Figure 2.12, with their centre at O_S and O_L. The radius of curve R_S is smaller than the radius of curve R_L. The two circular curves with different radii meet at a common point O_L. The compound curve is tangential to three straights AB, KM, and BC at T_1, N and T_2, respectively. Points N, O_S and O_L will lie in a straight line. The tangents AB and NK intersect at K, and tangents BC and NM will intersect at M.

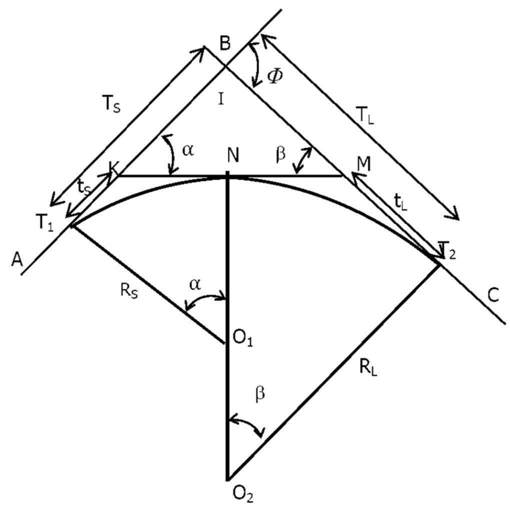

Figure 2.12 A compound curve

2.4.1 Elements of a compound curve

In Figure 2.12, it is shown that a compound curve has three straights AB, BC and KM which have tangential at T_1,T_2 and N, respectively. The two circular arcs T_1N and NT_2 having centres at O_1 and O_2. The arc having a smaller radius may be first or second curve. The tangents AB and BC intersect at point B, AB and KM at K and BC and KM at M.

If T_1 is the point of curvature, T_2 is the point of tangency, B is the point of intersection, N is the point of compound curve (PCC), T_S is the length of tangent of the first curve, T_L is the length of tangent of the second curve, t_s is the length of tangent to curve T_1N , t_L is the length of tangent to curve NT_2 , K is the vertex of the first curve, M is the vertex of the second curve, R_s is the smaller radius O_1T_1, R_L is the larger radius O_2T_2, Δ is the deflection angle between rear tangent (AB) and forward tangent (BC), α is the deflection angle between rear tangent (AB) and the common tangent (KM), β is the deflection angle between forward tangent (BC) and common tangent (KM), L_1 is the length of first chord, L_2 is the length of second chord, L is the length of long chord from T_1 to T_2, l_S is the length of first arc, l_L is the length of second arc, l is the length of total curve, R_s is the smaller radius O_1T_1, R_L is the larger radius O_2T_2, Φ is the deflection angle between rear tangent (AB) and forward tangent (BC), α is the deflection angle between rear tangent (AB) and the common tangent (KM), and β is the deflection angle between forward tangent (BC) and common tangent (KM), then-

Angle $T_1BT_2 = I = 180° - \Phi$

$\Phi = \alpha + \beta$ (2.26)

$KN = KT_1 = t_s = R_s \tan(\alpha/2)$ (2.27)

$MN = MT_2 = t_L = R_L \tan(\beta/2)$ (2.28)

$$KM = KN + NM = t_S + t_L = R_S \tan\frac{\alpha}{2} + R_L \tan\frac{\beta}{2} \quad (2.29)$$

Applying sine relationship in ΔBKM, we get-

$$\frac{BK}{\sin\beta} = \frac{KM}{\sin I}$$

$$\frac{BK}{\sin\beta} = \frac{KM}{\sin(180^0 - \phi)}$$

or $$\frac{BK}{\sin\beta} = \frac{KM}{\sin\phi}$$

so $$BK = \frac{KM \sin\beta}{\sin\phi} = \frac{(t_S + t_L)\sin\beta}{\sin\phi}$$

But $T_S = BT_1 = BK + KT_1$

$$= \frac{(t_S + t_L)\sin\beta}{\sin\phi} + t_s \quad (2.30)$$

Similarly, applying sine relationship in ΔBKM

$$BM = \frac{KM \sin\alpha}{\sin\phi} = \frac{(t_S + t_L)\sin\alpha}{\sin\phi}$$

$$T_L = BT_2 = BM + MT_2$$

$$= \frac{(KM)\sin\alpha}{\sin\phi} + t_L \quad (2.31)$$

Length of the first curve = $l_S = R_S\ \alpha\ (\pi / 180)$ (2.32)

Length of the second curve = $l_L = R_L\ \alpha\ (\pi / 180)$ (2.33)

Total length of curve (l) = $l_S + l_L$ (2.34)

2.4.2 Setting out the compound curve

(i) The compound curve may be set out by the method of deflection angles from two points T_1 and N; the first curve from point T_1 and the second one from point N.

(ii) Locate B, T_1 and T_2, and find out the chainage of T_1 from the known chainage of B and length BT_1.

(iii) Find out the chainage of F by adding the length of the first curve to the chainage of T_1, and find the chainage of T_2 by adding the length of the second curve to the chainage of F.

(iv) Calculate the deflection angles.

(v) Set up the theodolite at T_1, and set out the first curve.

(vi) Shift the instrument and set it at point F. With the horizontal angle set to $\left(360^0 - \frac{\alpha}{2}\right)$, take a back sight on T_1 and transit the telescope and swing through $\frac{\alpha}{2}$, the line of sight will be directed along the common tangent FE and the reading will read 360^0.

(vii) Set the vernier to the first deflection angle as calculated for the second curve, thus directing the line of sight to the first point on the second curve.

(viii) Process is repeated until the end of the second curve is reached.

Check: Measure the angle T_1FT_2, which must equal $180^0 - \frac{\phi}{2}$.

2.5 Reverse Curves

A curve consisting of two circular arcs of similar or different radii having their centres on opposite sides of the common tangent at the point of reverse curvature is known as a *reverse curve* (Figure 2.13). It is also known as a *serpentine curve* or *S-curve* due to its peculiar shape. It is generally used when two lines intersect at a very small angle. Reverse curves are used to connect two parallel roads or railway lines. These curves are best suited for hilly terrains and highways for relatively low-speed vehicles. Reverse curves are not advisable to use on the highways and railways which are meant for high-speed traffic movement because of the following reasons:

(a) A sudden change in direction can be dangerous for vehicles.

(b) A sudden change in curvature and direction increases wear & tear in vehicles, and also provides discomfort to the people traveling along the route.

(c) It may cause the vehicle to overturn over a reverse curve, if the vehicle is moving with a greater speed. careless.

(d) At the Point of Reverse Curvature (PRC), super-elevation can't be provided.

(e) Sudden change in super-elevation from one edge to another edge on reverse curve is required which is difficult to achieve.

(f) The curves cannot be properly provided superelevation at the point of reverse curvature

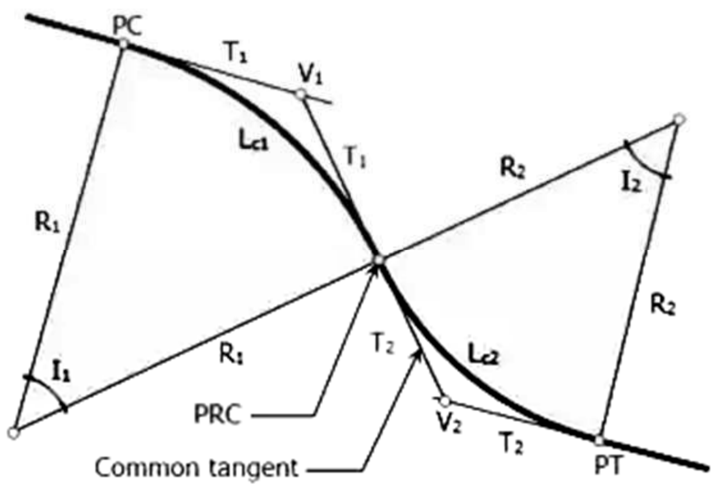

Figure 2.13 A reverse curve

If PC is the point of curvature, PT is the point of tangency, PRC is the point of reversed curvature, T_1 is the length of tangent of the first curve, T_2 is the length of tangent of the second curve, V_1 is the vertex of the first curve, V_2 is the vertex of the second curve, I_1 is the central angle of the first curve, I_2 is the central angle of the second curve, Lc_1 is the length of first curve, Lc_2 is the length of second curve, L_1 is the length of first chord, L_2 is the length of second chord, and $(T_1 + T_2)$ is the length of common tangent measured from V_1 to V_2, then-

Finding the chainage of PT:

(i) Given the chainage of *PC*, then the

Chainage of PT= Chainage of PC+Lc_1+Lc_2 (2.35)

(ii) Given the chainage of V_1, then the
Chainage of PT= Chainage of $V_1 - T_1 + Lc_1 + Lc_2$ (2.36)

2.5.1 Elements of a reverse curve

Figure 2.14 shows a reverse curve made up of two different radii. In this Figure, R_1 is the smaller radius ($O_1A=O_1D$), R_2 is the larger radius ($O_2D = O_2B$), Δ_1 is the angle subtended at the centre by the arc of smaller radius R_1, Δ_2 is the angle subtended at the centre by the arc of larger radius R_2, V is the perpendicular distance (AJ=MN) between two straights (parallel tangents) AM and BN, h is the distance between the perpendiculars at A and B, L is the length of the line joining the tangent points A and B.

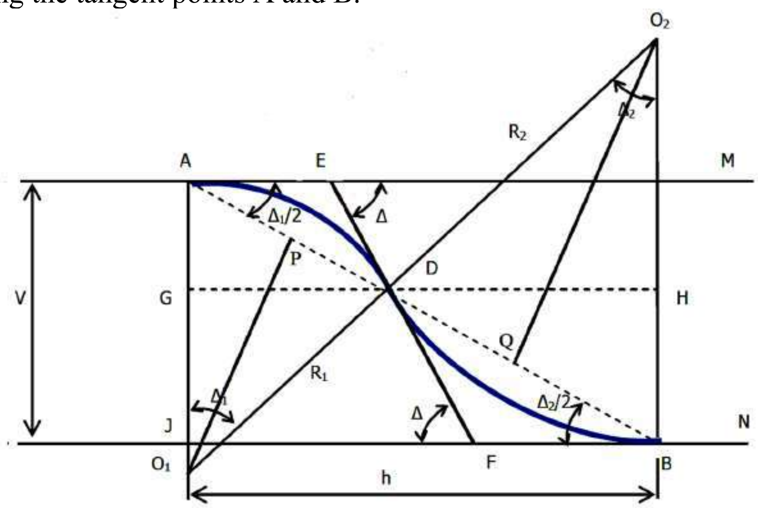

Figure 2.14 Elements of a reverse curve

D is the point of reverse curvature, and from it a line perpendicular to the straights AO_1 and BO_2 is drawn, cutting these at G and H. Draw perpendicular from O_1 and O_2 at line AB which will cut this line at P and Q, respectively, dividing the angle of deflection into half $\angle AO_1P = \angle DO_1P$, and $\angle BO_2Q = \angle DO_2Q$).

When $\Delta_1 = \Delta_2 = \Delta$
Perpendicular distance (V) = AG + GJ) = (AG + BH)
$V = (O_1A - O_1G) + (O_2B - O_2H)$ (2.37)
Here, $O_1A = R_1$, and $O_2B = R_2$
$\cos \Delta = O_1G / O_1D = O_1G / R_1$
$O1G = R_1 \cos \Delta$
$\cos \Delta = O_2H / O_2D = O_2H / R_2$
$O_2H = R_2 \cos \Delta$
From equation 2.37,
$V = (R_1 - R_1 \cos \Delta) + (R_2 - R_2 \cos \Delta)$
$V = R_1(1- \cos \Delta) + R_2 (1- \cos \Delta)$
$= (1- \cos \Delta)(R_1 + R_2)$
$V = (R_1 + R_2) \text{ versin } \Delta$ (2.38)

Total length (L) = AD + DB
But AD = AP + PD and DB = DQ + QB
In triangle O_1PA

$\sin \Delta/2 = AP / O_1A$
$AP = R_1 \sin \Delta/2$
$AD = 2 R_1 \sin \Delta/2$ (since AP = PD)
Similarly, in triangle O_2QB
$\sin \Delta/2 = BQ / O_2B$
$BQ = R_2 \sin \Delta/2$
$DB = 2R_2 \sin \Delta/2$ (since DQ = QB)
Total length (L) = AD + DB
So, $L = 2R_1 \sin \Delta/2 + 2R_2 \sin \Delta/2$
$L = 2(R_1 + R_2) \sin \Delta/2$ (2.39)

In triangle ABJ
$\sin \Delta/2 = AJ / AB = V / L$
Equations 2.39 can be written as-
$L = 2(R_1 + R_2) V / L$
$L^2 = 2V (R_1 + R_2)$
$L = \sqrt{[2V (R_1 + R_2)]}$ (2.40)

Distance between the end points of the reverse curve measured parallel to the straights (h) = GD + DH
In triangle GO_1D
$\sin \Delta = GD / O_1D$ or $GD = R_1 \sin \Delta$
In triangle BO_2D
$\sin \Delta = DH / O_2D$ or $DH= R_2 \sin \Delta$
So, h = GD + DH
$= R_1 \sin \Delta + R_2 \sin \Delta$
$h = (R_1 + R_2) \sin \Delta$ (2.41)

Length of the first curve AD (l_1) = $R_1 \Delta (\pi /180)$
Length of the second curve DB (l_2) = $R_2 \Delta (\pi /180)$
Total length of the curve ADB ($l_1 + l_2$) = $\Delta (\pi /180) (R_1 + R_2)$ (2.42)

If the radius of the two curves are equal-
From equation 2.38, $V = 2R \text{ versin } \Delta$ (2.43)
From equation 2.39, $L = 4R \sin \Delta/2$ (2.44)
or $R = L / (4 \sin \Delta/2)$
From equation 2.40, $L = \sqrt{(2V * 2R)} = \sqrt{4 VR}$ (2.45)
or $R = L^2 / 4V$
From equation 2.41, $h = 2R \sin \Delta$ (2.46)
From equation 2.42, total length of the curve ADB ($l_1 + l_2$) = $2R \Delta (\pi /180)$ (2.47)

2.6 Transition Curves

A transition curve is a non-circular curve of varying radius which is introduced between a straight and a circular curve for the purpose of giving ease in ride and change of direction along the route (Figure 2.15). The primary purpose of the transition curve is to enable vehicle moving at high speeds to make the change from the tangent section to the curves section, in a safe and comfortable fashion. When a vehicle enters or leaves a circular curve of finite radius, it is subject to an outward centrifugal force which can cause the shifting away of the passengers and the driver.

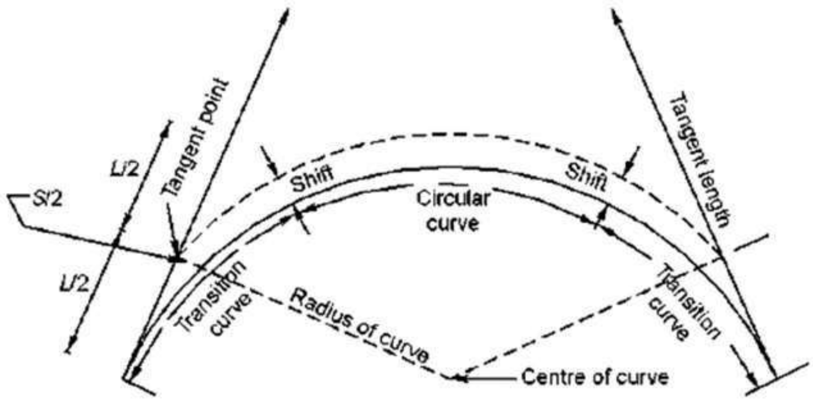

Figure 2.15 A typical transition curve

The transition curve can also be inserted between two branches of a compound or reverse curve. The transition from the straight line to the tangent to circular curve, and from the circular curve to the straight line should be gradual. The transition curve helps in obtaining a gradual increase of super-elevation from zero on the tangent to the required full amount on the circular curve. It avoids danger of derailment, side skidding or overturning or vehicles while moving, as well as avoids discomfort to the passengers.

The most common types of transition curves are shown in Figures 2.16 and explained below. There are three types of transition curves commonly used: (i) a cubic parabola, (ii) a cubical spiral, and (iii) a lemniscate. The first two are used on railways and highways both, while the third one is used on highways only.

(i) Cubic parabolic curve–In this curve, the rate of decrease of curvature is much low for deflection angles 4° to 9°, but beyond 9°, there is a rapid increase in the radius of curvature. It is mostly used in railways. The equation is:

$$y = x^3 / (6RL) \qquad (2.48)$$

Where y is the coordinate of any point, x is the distance measured along the tangent, R is the radius of curve, and $L$is the length of curve

(ii) Spiral curve– This an ideal transition curve. It is the most widely used curve as it can easily be set out with its rectangular co-ordinates. The radius of this curve is inversely proportional to length traversed. Hence, the rate of change of acceleration in this curve is uniform throughout its length. It is mostly used in railways.

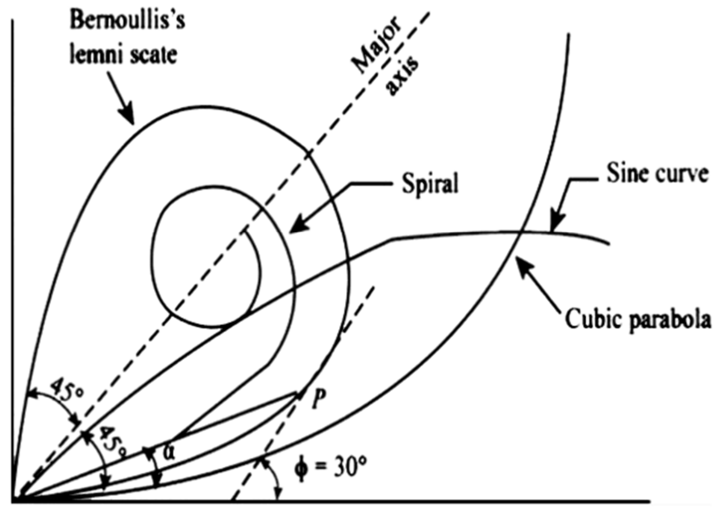

Figure 2.16 Various types of transition curves

(iii) Lemniscate curve– In this transition curve, radius decreases as the length increases, and hence there is a slight fall of the rate of gain of radial acceleration. It is mostly used in highways. It can be represented by the Bernoulli's lemniscate curve:

$$L = k \sqrt{(\sin 2\alpha)} \tag{2.49}$$

Where L is the length of polar distance of any point in meters, α is the polar deflection angle of that point in radians, and k is a constant

A transition curve should fulfil the following conditions:

(i) If should tangentially meet the tangent line as well as the circular curve.
(ii) The curve should have infinite radius (i.e., zero curvature) at the origin.
(iii) The rate of increase of curvature along the transition curve should be the same as that of increase of super-elevation.
(iv) The length of the transition curve should be such that the full super-elevation is attained at the junction with the circular curve.
(v) Its radius at the junction with the circular curve is equal to that of circular curve.

2.6.1 Super-elevation or Cant

When a vehicle passes from a straight line to a curve line, in addition to its own weight, a centrifugal force acts on it, as shown in Figure 2.17. Both the forces act through the centre of gravity of vehicle. The centrifugal force acts horizontally and tends to push the vehicle away from the centre or road. This is because there is no component force to counter balance this centrifugal force. To counteract this effect, outer edge of the curve is elevated or raised by a small amount as compared to the inner one. This raising of the outer edge of curve is called *super-elevation* or *cant.* The amount of super-elevation will depend upon on several factors, such as the speed of the vehicle and radius of the curve.