Hydraulic Engineering
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Lecture – 03
Basics of fluid mechanics - I (Contd.)

Welcome back to lecture 3, the topic for today is fluid statics 1. So, this is also the part of the basic fluid mechanics that we have been doing in the last 2 lectures to get started.

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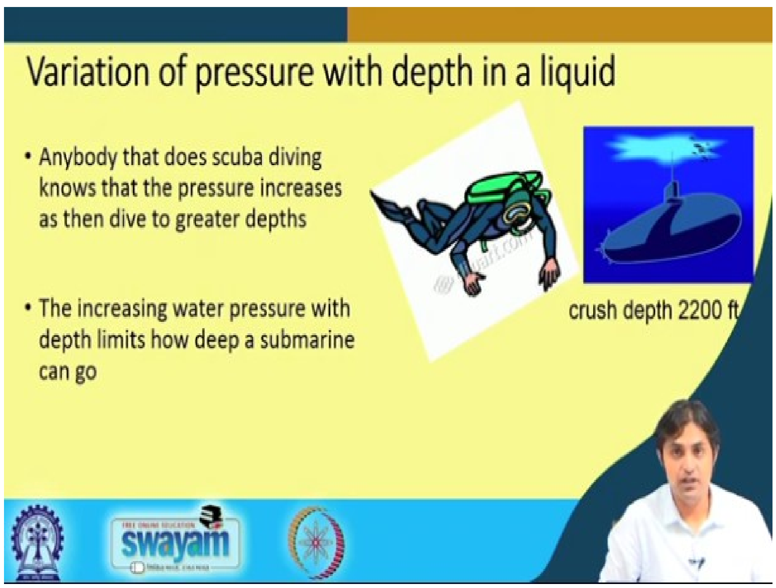

One of the most important question is the variation of pressure with depth in a liquid. How does the pressure vary? So, to be able to, you know, give a real feel has anyone of you have done scuba diving that you will observe that the pressure increases as then you go down. So, compared to if you are at the upper surface, than at the lower surface, at the lower depths the pressure will increase. The increasing water pressure with depth limits how deep a submarine can go.

These are the 2 classical example of the variation of pressure with depth and a liquid. Many of you might have observed this for in this case, the crush depth of 2200 feet is for this submarine. So, if the submarine goes below that due to the pressure of the water above the submarine, the submarine can, you know, crush.

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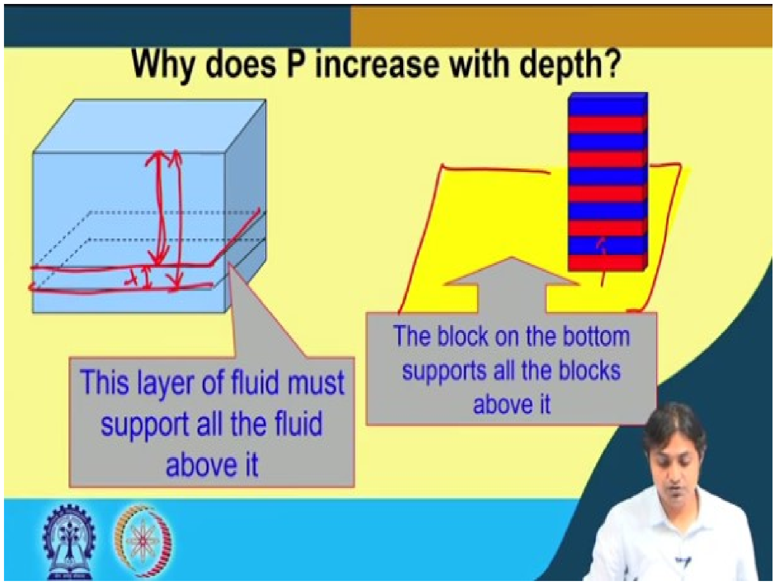

So, now the important question is why P increases with depth. This is a volume of liquid. As you can see there are some layers on it. So, this is one layer, right? This is one layer. So, this means, this particular layer that have to support the entire liquid that is above this layer and the same is for this layer. So, this means, this layer will have to support this much plus this much. So, this is how you can understand, why does pressure increases with depth, this layer of fluid must support all the fluid that is above it.

So, this is, this block the one that is shown in yellow in color shall be supporting all the blocks that are above it that is why, and another way to see why the pressure should increase with the depth.

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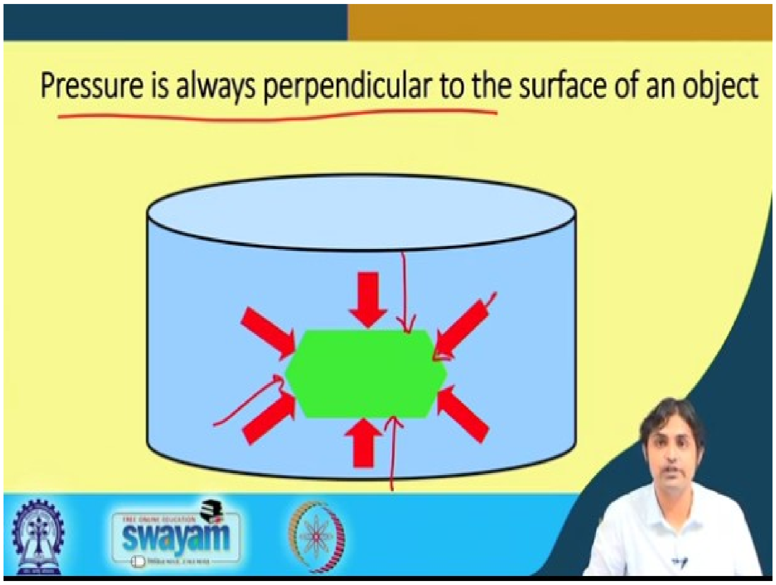

One of the other important feature of pressure is that pressure is always perpendicular to the surface, this is very important as you can see, if this is a surface we have a perpendicular like this, on this surface like this, on this surface like this, on this surface like this. Here we have already indicated by the red thick red arrows here.

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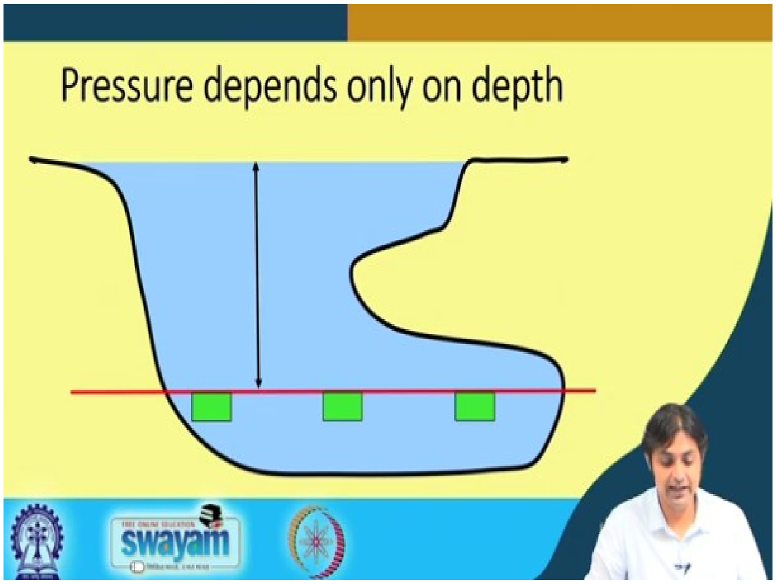

And the pressure does depend only on depth we are going to see that will do some basic derivation later in this lecture, where you will see that the dependence of pressure is only on depth is very important to note.

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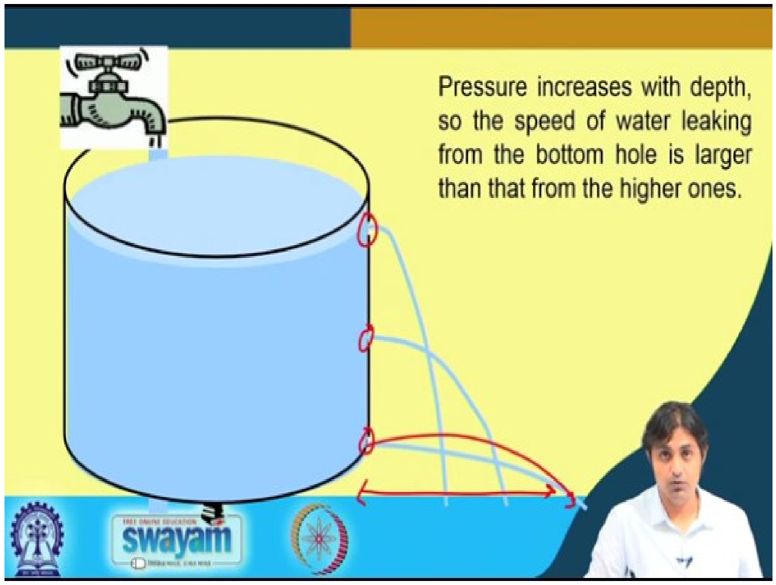

See, this is one of the animated examples which everybody of you must have encountered in your real life. So, there is a tap and this is a tank with some holes in it or some stopper valve. The valves are here, here, and here. So, we start filling the water. Let us suppose water gets filled. If we open this one this valve, you see the water trajectory is going to be something like this. Similarly, the one above it will have a trajectory like this.

The third one will have another trajectory like this, what does it say the distance that each one of them is covering is decreasing as the valve is going up. This means that the pressure increases with depth. Therefore, the speed of water leaking from the bottom hole is larger than the one at higher holes, because larger the pressure larger the force, therefore, higher the speed in layman's term. This is for you to understand the, I mean basic concepts of how this pressure system works.

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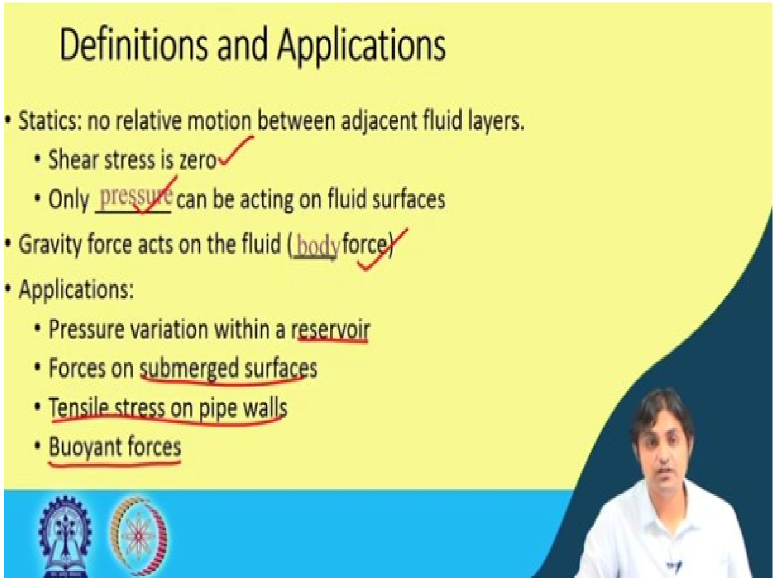

Now, some definitions and applications, in fluid statics as we have seen there is no relative motion between the adjacent fluid layers, therefore, the shear stress is 0 as we have already seen this in our previous lecture. So, what can be acting on the fluid surface? So, only pressure can be acting in that case on the fluid surface there is no shear only pressure forces. Gravity force acts on the fluid and that is called the body force.

So, gravity force when it acts in the body it is defined as the body force. What will be the applications of this particular thing that we are studying about pressure? We can know the pressure variation within a reservoir. So, it find application in reservoir, we are also able to find forces on submerged surface you will see many cases in real life where the surfaces are submerged. We are also able to find using this app this principle the tensile stress on pipe walls and also we are going to see the buoyant forces because buoyancy plays an important role in this.

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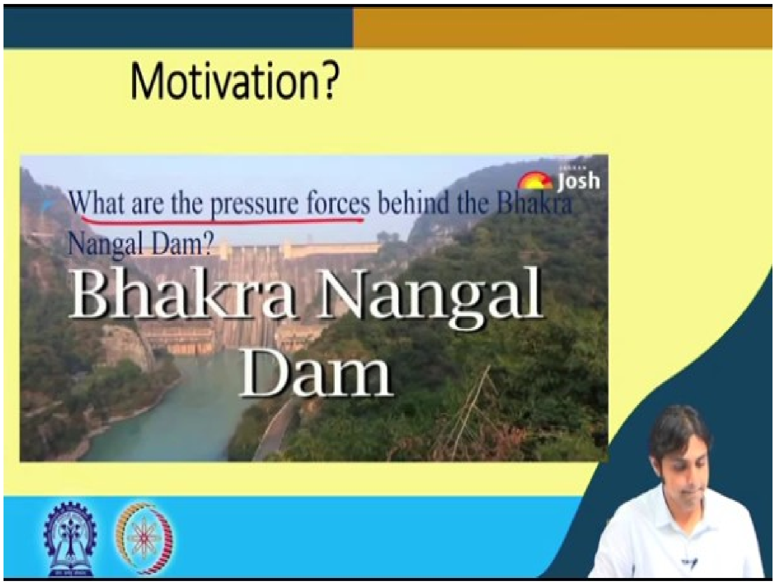

What actually is the motivation, the motivations are enormous as an Indian, we know we have all heard of Bhakra Nangal Dam. So, the forces acting on the Bhakra Nangal Dam, this is one of the very important thing and for you as a student, I would consider this as the biggest motivation design of dams for example. So, the question is, what are the pressure forces behind the Bhakra Nangal Dam? If this is the first step towards any design, you need to determine the forces or the parameters that determine the stability.

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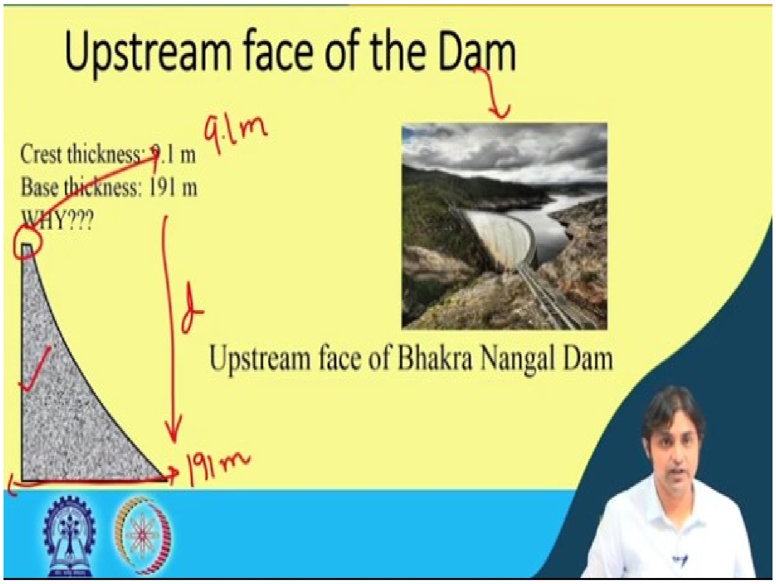

So, this is what I have shown here is the upstream face of the dam here. And the if you see it, if you put it in the 2 dimension this is what the Bhakra Nangal Dam looks like. So, this one here the crest thickness, this is the crest, this is only 9.1-meter, 9.1 meter, the however the base thickness is 191 meter. And why that happens? It is very simple to explain that as you go deeper and deeper.

So, this represents the increasing water depth in this direction. So, as you go deeper and deeper the weight of the water that it needs to support increases therefore, the base of to resist the force, the base of the dam needs to be larger. This is a very, very simple explanation why we have a smaller crest and base thickness is very high.

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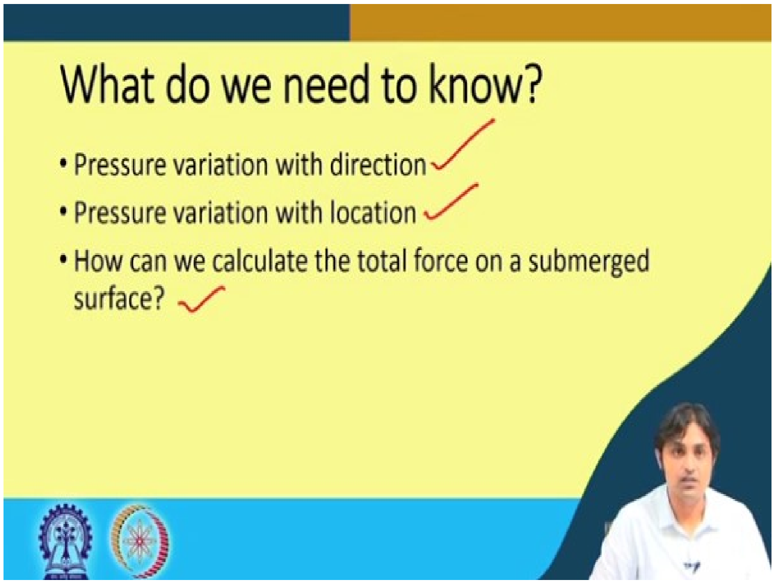

What do we need to know in while studying about pressure, we need to know the variation of pressure with direction, we also need to know pressure variation with the location. Direction is say at in what angle θ for example, location is at what depth for example, and how are we able to calculate the total force on the submerged surface. So, these are the 3 questions that we are going to answer today.

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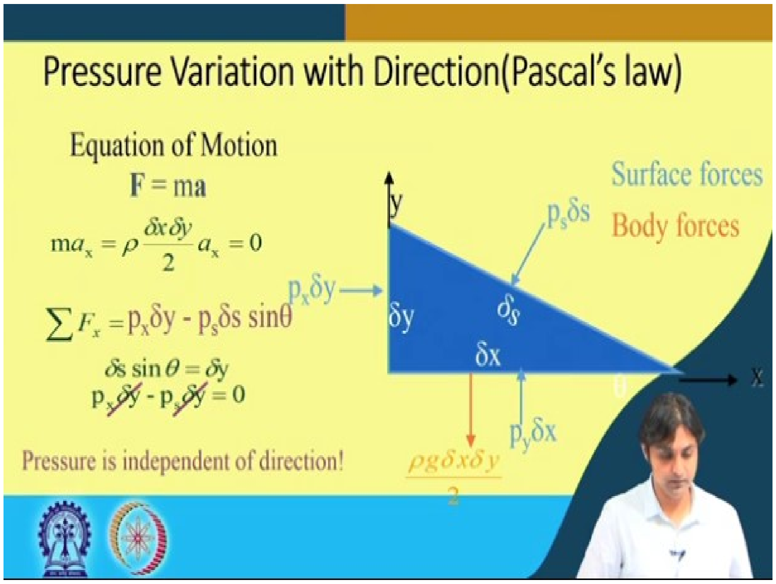

So, first we are going to see the Pascal's law, which is pressure variation with direction, very simple. This element a triangular element that is shown here this is a fluid element, this the length in x direction is δx, this angle is θ here, the y direction the length is δy and δ is the length of this surface. What we say first when we make a body diagram, we need to have the forces. So, the body force is of course, we talked about gravity. So, it is going to be the weight. What is going to be the weight? Very simple, its mass into $m*g$, the mass can be written as $\rho*V$. So, assuming in z direction, we have a unit length. So the area of this triangle is $\frac{1}{2}\delta x\delta y*1$, one will give us volume. So the mass is going to be $m=\rho*g*V$. So this is the weight of the element. Now looking at the surface forces, there will be surface forces acting what we have seen before, surface forces in case of fluid statics will be only due to the pressure.

So pressure will exert force on all the surfaces. So these 3 this one, this one, and this one are going to exert force on this surfaces, what is the magnitude, so, if there is the force on this the pressure is P_s it will be acting on this side. In a unit z direction so it will be P_s into area that is δS into 1 similarly in x direction it is going to be P_x if P_x is the pressure acting in this direction. And multiplied by δy into 1 and in this direction similarly it is going to be P_y into $\delta x*1$ because we have assumed, unit length in z direction.

Force is given by mass into acceleration $F=ma$. Therefore, mass into acceleration, mass is $m=\rho*V$. So $\frac{1}{2}\delta x\delta y\delta*1$ because we assumed unit an a_x, this should be equal to 0. And the summation of the forces on this body in the y direction is going to be P_x, this one δy, there will be a component in this direction for P_s δS this will be given by $P_s * \delta S * sin\theta$. So, this is this component here. If we also know that $\delta S.sin\theta$ is equivalent to δy because of trigonometry.

Therefore, if we put $\delta S.sin\theta$ as δy here we are going to get P_x δy - P_s δy. Very it is very obvious now, that the δy and δy should get canceled if they are not equal to 0. This give us $P_x = P_s$. Ok, this indicates that pressure is independent of the direction and this was one of the properties that we have seen in the beginning that pressure depends only upon depth, there is no directional influence on the variation of pressure. Great so, I will erase all ink so that, you are able to follow that in much more detail.

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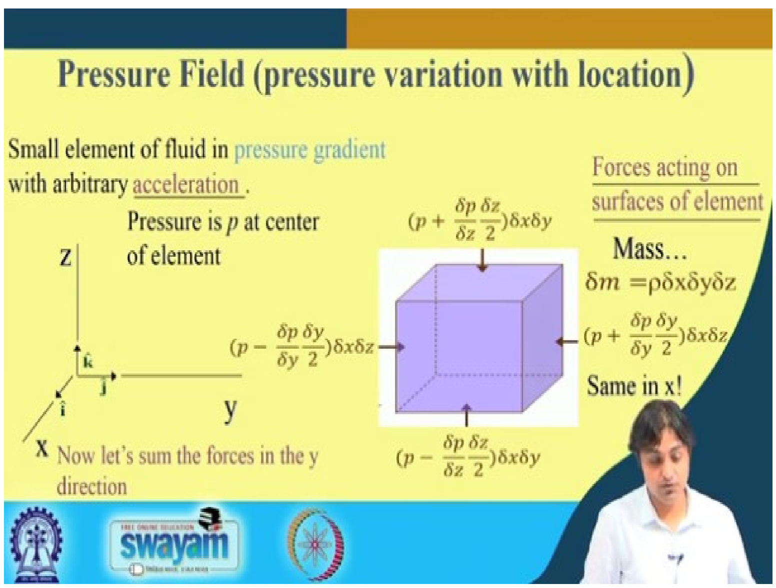

So, then as we said we should also be able to understand the pressure variation with location. So, the small element of we assume a small element of fluid in pressure gradient with an arbitrary acceleration, we assume, that is, what assumption is we define our axis x in this direction. So, this is the x direction, this is the y direction and this is z direction, we are assuming up mass of fluid in cubical format, this is a fluid element.

So, as we have done always what we are going to do is we are going to write the forces, but first assume that at the center, so, I am going to use the pen now, the pressure at this center is *P*. Therefore, this will now actually we will be able to help in the derivation and there is a uniform pressure gradient that is what our assumption is. So, we should be knowing the forces acting on the surface of the element. So, if the pressure at the center is *P* and then the pressure variation can be given in this direction as $\frac{\delta p}{\delta y}$.

So, this is pressure variation in y direction and what is the length this is *y / 2*. So, in this direction it will be $p - \frac{\delta p}{\delta y} * \frac{\delta y}{2}$. So, this is going to be the P_1 for example, $P_1 = p - \frac{\delta p}{\delta y} * \frac{\delta y}{2}$ and it should be multiplied by the area this is any way as a 3 dimension element. So, this area that is it is going to is $\delta x \delta z$. So, this is the force on this side.

Similarly, on the other side if the pressure is *p* here, and the gradient in if we are subtracting when we are going backwards which will be added when we are going in the forward direction and $\delta x \delta z$. Similarly, for this surface, this will be because this is z direction. So, it will be $p - \frac{\delta p}{\delta z} * \frac{\delta z}{2}$ to have the total pressure at this surface and overall multiplied by area this will give us a force and similarly in this direction as well, because we have assumed that the pressure increases in the upward direction.

So, well going down it should decrease its

$$(p - \frac{\delta p}{\delta z} * \frac{\delta z}{2}) * \delta x \delta y$$

I think this now, you should be able to write on your own these terms. So, what the mass of the element is going to be. Very simple, the delta m is going to be density multiplied by Volume. Volume is very simple $\delta x \delta z \delta y$. So, we have now written most of the things in one of these directions.

We can see we have assumed in the direction of y this, the same can be done in x also, but we are going to concentrate in one direction right now, y the variation of pressure in y direction for example. So, as we have already written the forces here in all the 4 sites, we are going to do the force balance here. So, as I said I will try to erase. So, that you are able to, great.

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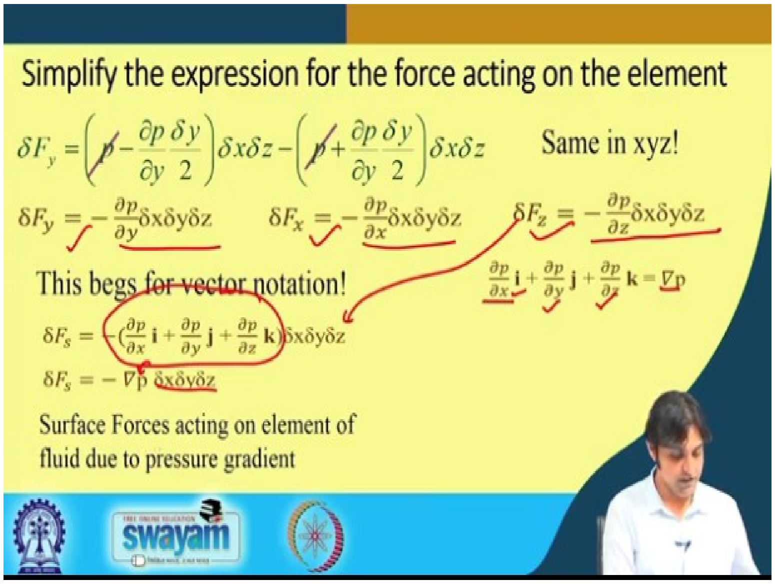

So, now, we have to simplify the expression for the force acting on the element, very simple. So, in the y direction, the force is going to be this was the force, pressure force minus these forces. This was acting in this side and this force was acting from the right. So, p and p can get canceled, we will have the same in X, Y and Z. So I will just erase this one. So, the summation of δF_y , I mean, if you do this addition, it will be $\delta F_y = -\frac{\delta p}{\delta y}\delta x \delta z \delta y$.

Similarly, if we do as I said we can do the same in x direction, we are going to get $\delta F_y = -\frac{\delta p}{\delta x}\delta x \delta z \delta y$ this is important. And we can also get the same thing in z direction. So, it is going to be $\delta F_z = -\frac{\delta p}{\delta z}\delta x \delta z \delta y$ it is just the force that we have calculated the differential force because of pressure gradient. Now, we can write these forces in vectors.

So, if you see the pressure can be written as $\nabla p = \frac{\partial p}{\partial x}\,i + \frac{\partial p}{\partial y}\,j + \frac{\partial p}{\partial z}\,k$. So, the summation of all the forces here, summation of all the forces, because this delta F_y is in y direction Fx in x direction, and F_z in z direction, we can write the total force. If you sum these three 1,2,3, you are going to get this equation which is $\delta F_s = -(\frac{\partial p}{\partial x}\,i + \frac{\partial p}{\partial y}\,j + \frac{\partial p}{\partial z}\,k)\delta x \delta z \delta y$.

So, delta F_s can be written as we can write this entire term as ∇p this is a standard mathematical notation and we keep this $\delta x \delta z \delta y$ as it is $\delta F_s = -\nabla p\ \delta x \delta z \delta y$. Now, the surface forces acting on the element of fluid due to pressure gradient. So this is the surface forces acting on element of the fluid due to the pressure gradient.

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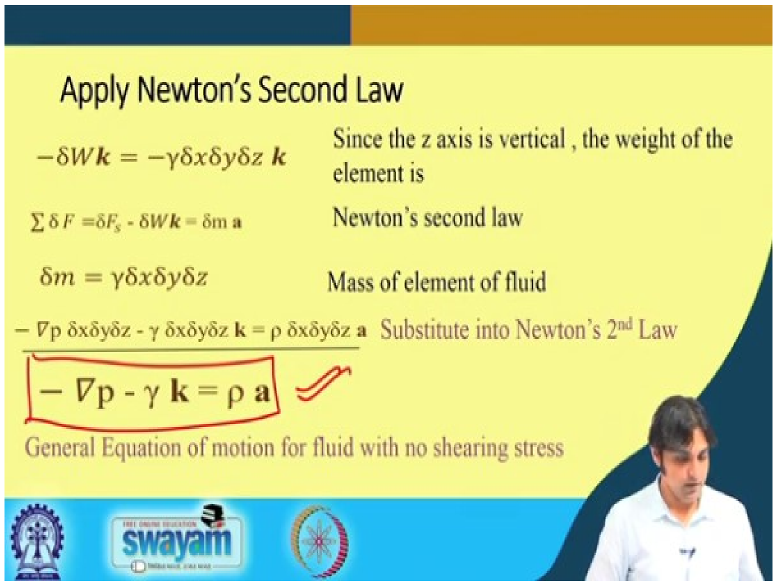

So, what happens? Now we have to apply the Newton's second law. So in z direction, if you see there is weight acting, which is balanced in the k direction weight can be written as $\gamma = \rho g$, and this is the volume. So, weight is mass, into gravity into, so, mass can be written as $\delta m = \rho\ \delta x \delta z \delta y$. Since z axis is vertical, the weight of this element can be written as, see there is a force component, so, the net force will be the surface forces plus the weight since the weight is acting downward we have assumed a negative sign here and this will be equal to $\delta m . a$.

So, net force $\delta F = m.a$, we are doing just the Newton's second law here. So, mass as we already seen is $\delta m = \rho\ \delta x \delta y \delta z$ this should be ρ, this is mass of the element of the fluid. So, the net force because there is no acceleration, so, the net force that has come to be

$$-\nabla p\ \delta x \delta z \delta y - \gamma \delta x \delta z \delta y = \rho\ \delta x \delta y \delta z\ a$$

, this is the weight and this is the pressure forces. This is the weight and this you will see this is acceleration mass into acceleration. This is surface force. This is body force.

So, now substitute into Newton's second law, if you see we can get $\delta x \delta z \delta y$ canceled from everywhere. So, the final equation, so that you are able to see I will erase all ink here will be. So, this is the general equation of motion for fluid with no shearing stress this is very very important equation you should remember this. Great.

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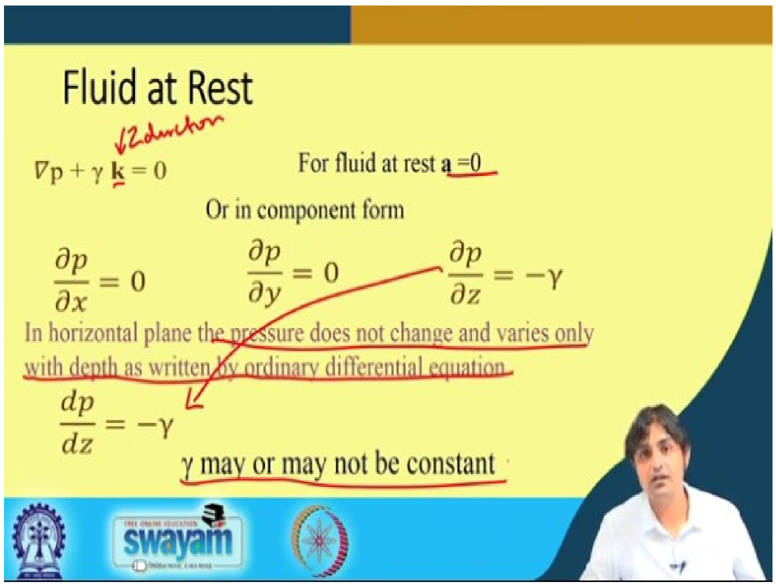

Now, as you see this equation here,

$$-\nabla p - \gamma k = \rho a$$

, one of the special cases is when the fluid is at rest. So, in case of fluid at rest the acceleration should be 0. And if we put a is equal to 0 in the previous equation we can get $\nabla p + \gamma k$, k this is the direction is equal to 0. Or if you write in component form, we can write δp because this is this is z direction.

So, in x a direction there is only pressure. So, $\frac{\delta p}{\delta x} = 0$, $\frac{\delta p}{\delta y} = 0$, but in z direction it will be $\frac{\delta p}{\delta z} = -\gamma$. In horizontal plane, the pressure, so, what does it mean in horizontal plane the pressure does not change and varies only with depth as written by ordinary differential equation. So, in case of no acceleration there will be no pressure variation, in x and y direction only in z.

So, this equation z direction can be written as $\frac{dp}{dz} = -\gamma$. Now, one of the important thing is γ may or may not be constant and that is what we are going to see next.

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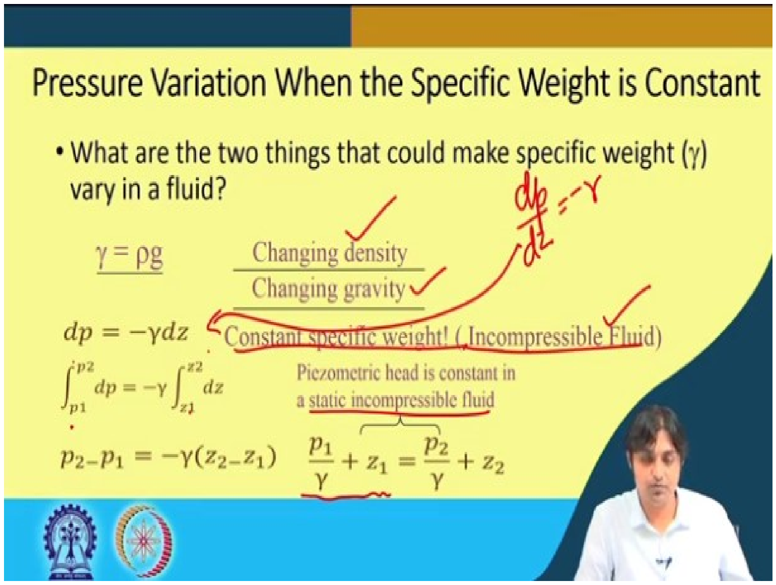

What are the 2 things that could make specific weight γ vary in a fluid? That is, very simple because γ you already know gamma is written $\gamma = \rho g$. So the 2 things are if the change in density is changing or if the gravity is changing, there these 2 I mean that can happen if you go to higher elevations both can be affected the density gravity if you go on from one latitude to the other, you might have read from your physics class that if you go to some place some other place on Earth, gravity might be a little different, not very different from but we wish consider all those cases.

So, if we can write the previous equation where the previous equation was what $\frac{dp}{dz} = -\gamma$. So this is what we exactly written, we have took dz on this side. Let us assume that the specific weight is constant and this happens for this type of fluid which is incompressible fluid. So ρ is constant and under assumption of gravity also does not change this type of fluid is called incompressible fluid.

We will be dealing with incompressible fluid throughout this course. So, if it is incompressible fluid we can simply integrate from p1 to p2 and from z1 to z2 because γ is constant, we can take this out of the integration and we can simply write to $p2 - p1 = -\gamma\,(z2 - z1)$. So, rearranging will give this one $\frac{p_1}{\gamma} + Z_1 = \frac{p_2}{\gamma} + Z_2$. So, this is called piezometric head and this head is constant in a static incompressible fluid that is one important take from this slide.

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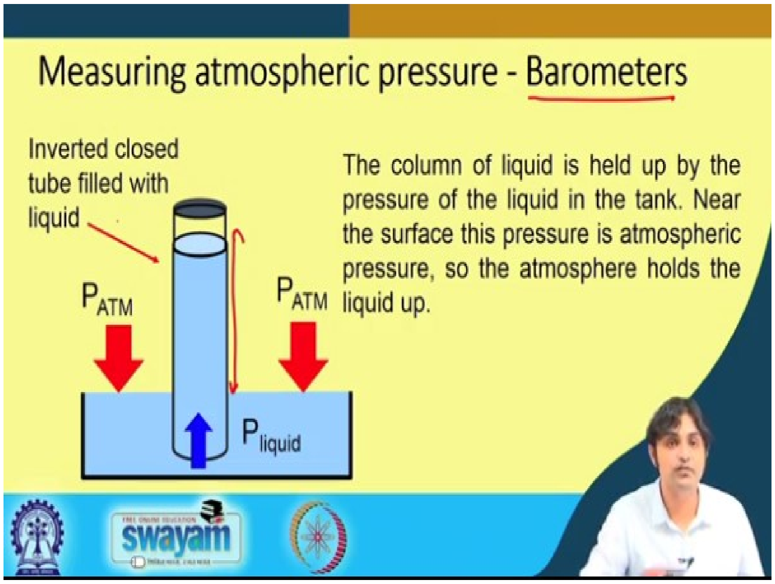

An important question is, how do we measure atmospheric pressure. One of the simplest devices the barometers. See, what happens is that the column of liquid is held up by the pressure of the liquid in the tank. Near the surface this pressure is atmospheric pressure, so the atmosphere holds the liquid up. So, this which I mean this liquid is being supported by the atmospheric pressure, this is an inverted closed tube filled with liquid.

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