Hydraulic Engineering
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Lecture-11
Basics of fluid mechanics-II Contd.,)

Welcome students, this is going to be the last lecture for the basics of fluid mechanics 2. Where we are going to see the conservation of momentum in more detail.

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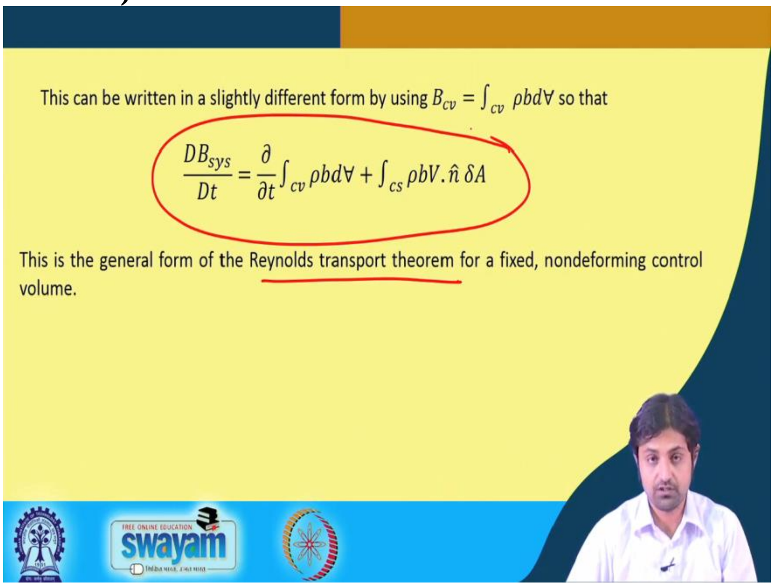

So, starting with the last slide, where we left in the last lecture and the we derived the general form of Reynolds transport theorem here. So, this is the general form and now in the upcoming lecture and slides, what we are going to do is we will apply this Reynolds transport theorem for derivation of different conservation equations.

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So, now we are moving from a system to a control volume. So, we will see, how when we apply B system to mass, what is going to happen. We will see about linear momentum, we will just it can also be actually applied to moment of momentum, we are not going to cover that. This principle can also be applied to the energy conservation which we are also not going to see in this lecture.

So, we are concentrating mostly on mass and linear momentum because this is of maximum use in the upcoming regular lectures and hydraulic engineering course. So, I mean, the idea is that we put it all together this basics of fluid mechanics 2 and proceed to our regular lectures on hydraulic engineering.

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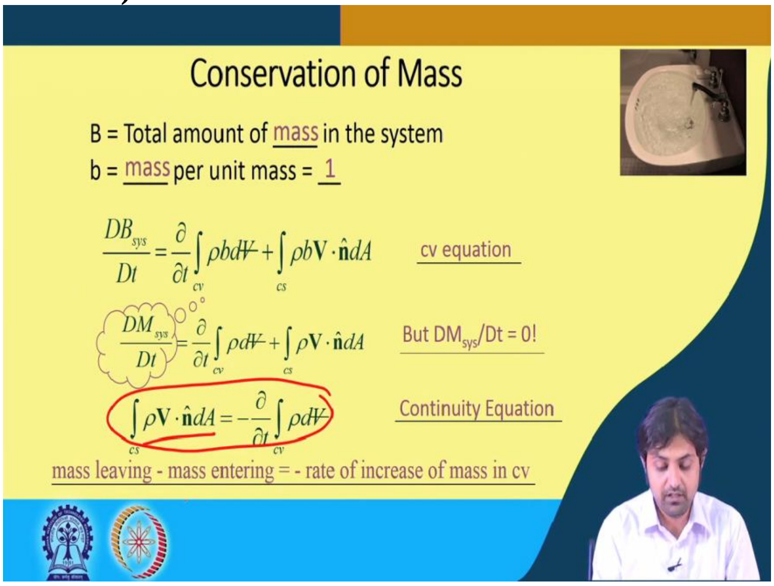

So, now conservation of mass, how can we apply, what we have learned in Reynolds transport theorem to the conservation of mass. So, B is total amount of mass in the system, when we do the conservation of mass B which we saw in Reynolds transport theorem is the total amount of mass in the system and b will be mass per unit mass which is equal to 1. So, we have determined B and b for the conservation of mass. So, B is M, capital B is M and small b is 1.

So, this is the general form of Reynolds transport equation that we have derived in the previous lecture and the also we saw this in the first slide of today's lecture. So, this is the control volume equation for the general Reynolds transport theorem. And what we have done here is we have equated B system to mass of the system. As b is 1 this b is this simply this term becomes ρ dv and this also b here is 1 so, it becomes a ρ V dot n cap d A.

So, what is this, we know that in the conservation of mass the total mass of the system with respect to the time DM, $DM_{sys}/Dt = 0$. So, we can simply write in the control volume

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = -\frac{\partial}{\partial t}\int_{cv} \rho d\forall$$

.This is the equation that we get by substituting B system as mass B as mass system and b as 1 as we have seen. So, this is actually nothing but a continuity equation.

So, I will just erase all the ink. So, it says mass leaving minus mass entering you remember, we saw the property influx was the property leaving minus property entering, here in this case is mass is equal to rate of increase of mass in the control volume. This equation here, tells us that mass leaving minus mass entering is minus rate of increase of mass in the control volume.

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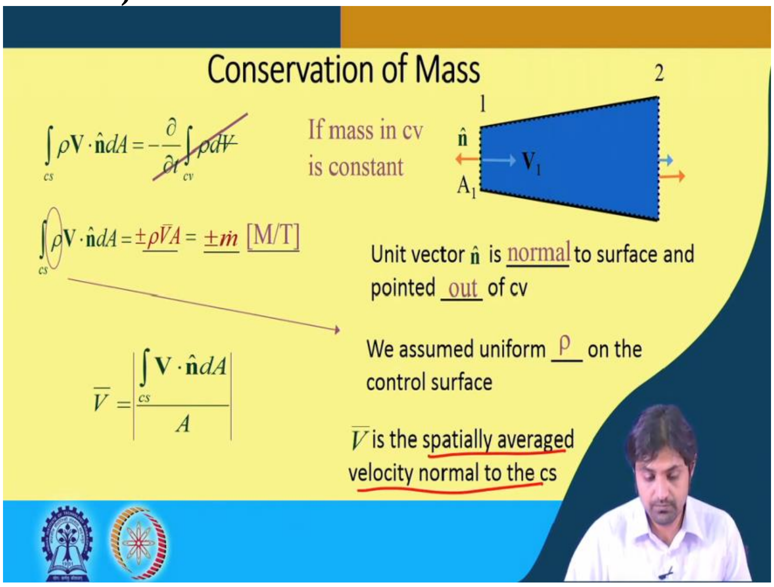

So, now, we are going to see, so, unit vector n cap here is normal to the surface. So, this is the surface 1 which this is normal and on the surface 2 it is in this direction. So, as already indicated by the red arrows, and is pointed out of the control volume in both the cross sections whether it is 1 or 2, the unit vector is normal to the surface and pointed out of the control volume.

So, if the mass in the control volume is constant what we can say

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = -\frac{\partial}{\partial t}\int_{cv} \rho d\forall$$

from the last slide. So, there is no rate of change of mass, if we assume, the mass in the control volume is constant, then this gives ρ v dot n cap dA is equal to, so, here we assumed, uniform ρ on the control surface in this one, so, ρ can be taken out. So, we can simply write for this particular case

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \pm \dot{m}$$

So, we can simply write the velocity, average velocity as here this quantity integral V dot n cap dA divided by the whole area of cross section. So, V bar is the spatially averaged velocity normal to the cross section, this is important to note.

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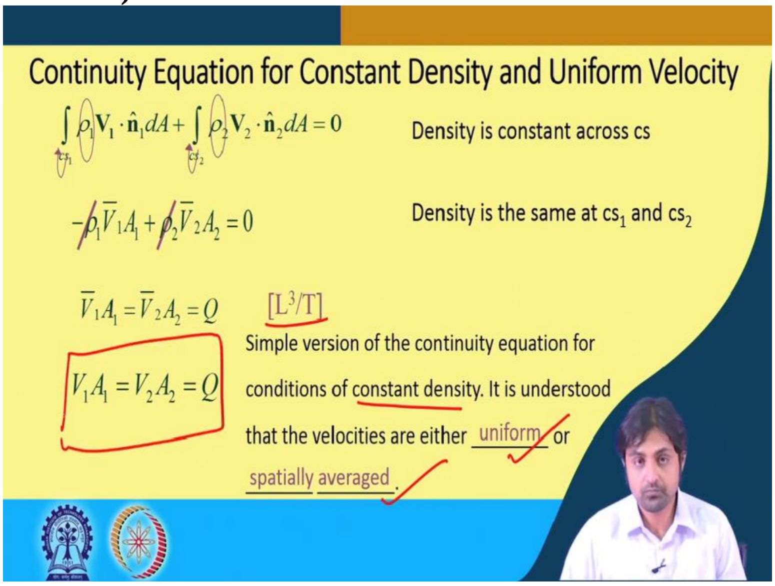

So, now continuity equation for constant density and uniform velocity, so, if there are 2 cross sections as we have seen in this, you know, this problem. We can simply write

$$\int_{cs_1} \rho_1 \mathbf{V}_1 \cdot \hat{\mathbf{n}}_1 dA + \int_{cs_2} \rho_2 \mathbf{V}_2 \cdot \hat{\mathbf{n}}_2 dA = 0$$

So, density is constant across this entire cross section. So, this we can take out as indicated by these arrows here. So, this one, can be simply written as, because the vector here was pointing outward.

So, if we take this direction as positive, this n 1 cap will be negative, I mean, n the vector that will be negative sign, as shown here

$$-\rho_1 \overline{V}_1 A_1 + \rho_2 \overline{V}_2 A_2 = 0$$

, because the density same at the cross section 1 and cross section 2. So, ρ 1 ρ 2 we can take out because it is the same. So, this gives us

$$\overline{V}_1 A_1 = \overline{V}_2 A_2 = Q$$

. So, using our Reynolds transport theorem or RTT abbreviated we obtained equation of continuity. This is same as we saw in fluid kinematics.

So, I am going to, you know, erase all the ink and this has a dimension of litre cube per length cube by time, not litre cube, length cube by time. So, V 1 A 1 is equal to V 2 A 2 is equal to Q. This is the simple version of the continuity equation for conditions of constant density. It is understood that the velocities are either uniform or Spatially averaged in both cases this is the equation of continuity.

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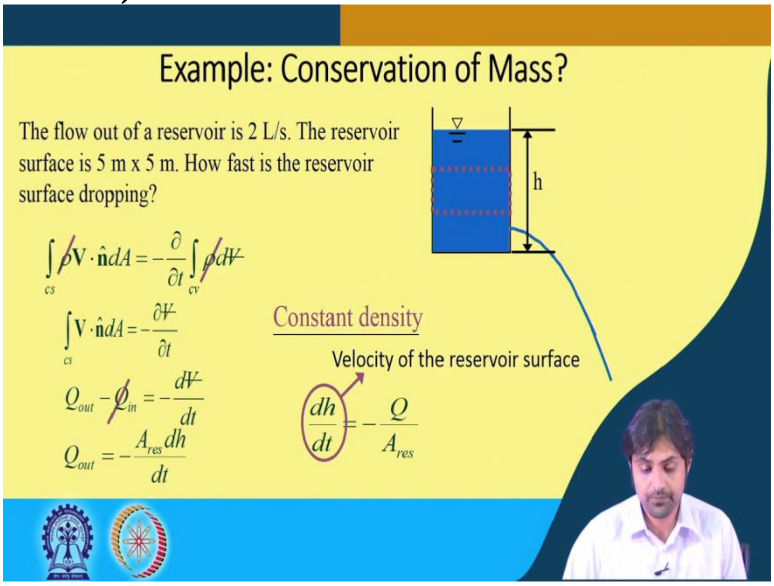

So, some examples of conservation of mass, the flow out of reservoir, for example, is 2 liters per second. The reservoir surface is 5 meters into 5 meter, the question is how fast is the reservoir surface dropping. So, you see the control surface that we have drawn here, this is the entire height h, as already indicated. So, the equation of continuity says from Reynolds transport theorem ρ V dot n cap dA is equal to - del / del t of ρ dv. Because the total mass of the system the control volume is not changing.

That is why we arrived at this equation. So, ρ because it has the same density. So, we can cancel these out. So, we get V dot n cap dA is equal to - del V del t . That is the entire volume, if we integrate the volume the entire volume. So, or these can say Q out - Q in that is the flux leaving minus the dis flux coming in is equal to - dV dt. There is no nothing that is entering. So, Q in is 0 so, Q out will be simply - area of reservoir into d h / dt or we can simply write the rate of fall of height with respect to time is this - Q / area of the reservoir.

So, dh / dt is the velocity of the reservoir surface. So, this is what we obtain, we can substitute Q is given 2 liters per second, we can convert it into SI unit and area of the reservoir, we can calculate, because it is rectangular in shape. So, this is also known. So, we are able to calculate dh / dt. I think, I will leave this up to you to do the numerical to derive the numerical values, which is the most simplest that you are able to find.

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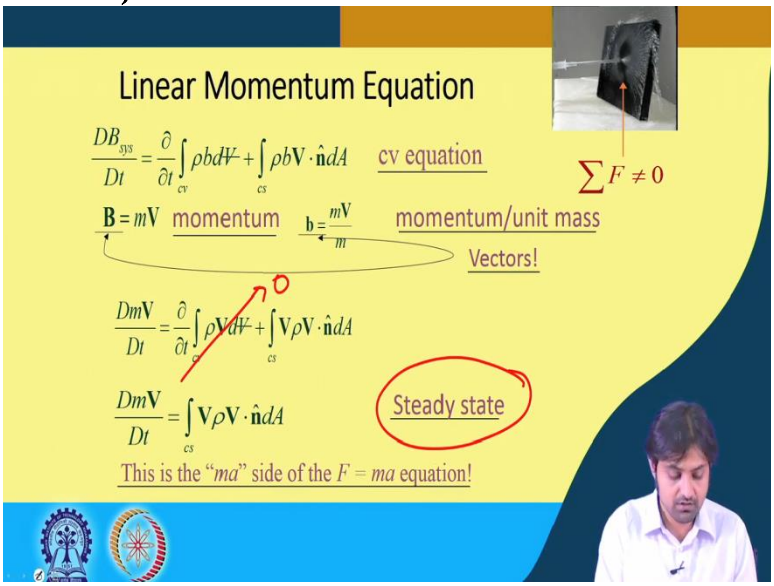

So, now the linear momentum equation. So, an example here is, if you see this figure here, this is so, suppose if you shoot water jet onto a wall for example, this is one of this example where, you know, linear momentum is associated, there is a velocity with which water is approaching and when it touches this wall it comes to rest. So, here we can actually apply the momentum conservation linear momentum. So, what happens is, as I said we are going to use the same concept of Reynolds transport theorem for the linear momentum equation too.

Here, I mean, in this particular case, the net force is not going to be 0 because this momentum before hitting the certain momentum and after hitting it turns to 0. So, the rate of change of momentum is force and the rate of change of momentum is not 0. Therefore, force will not be going, I mean, there will be some finite force. So, coming back to our Reynolds transport equation, here, what we do is so, this is the control volume equation, as I told you from the Reynolds transport theorem.

So, what we do we say B in our case when we want to derive the linear momentum equation B should be mass into velocity or called momentum. This is very normal term, I mean, in fluid or any type of mechanics, mass into velocity is momentum. I will just take this away. And b by definition will be the property divided by mass. So, it will be m V as shown here m V / m, this can get cancelled, so it is momentum per unit mass. So, be careful that these are actually vectors.

So, now coming back to substituting this B here and b, you know, here, so, substituting this here and b here and here, this is the equation that we obtain. So, this equation it is going to be D m V Dt is equal to ρ V dv dot n cap dA in for steady state, because then if it is steady state, this term will go to 0 for the steady state because there is no change in property with respect to time. So, del del t of anything is going to be 0 for steady state solution. So, this is the ma side of f is equal to ma equation. This is the mass into acceleration side of f = ma equation.

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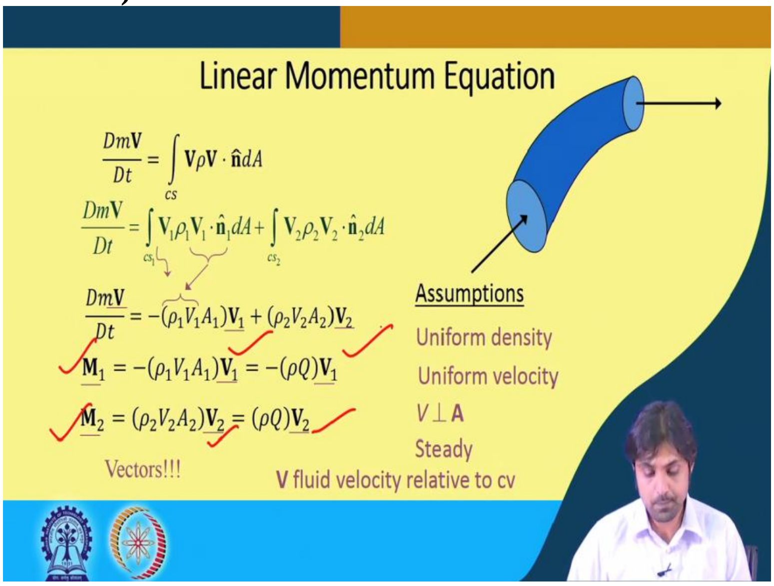

So, now, we similar to the mass we assume cross sections here and we use for the steady state equation , the equation that we have obtained in this slide on the next page. So,

$$\frac{Dm\mathbf{V}}{Dt} = \int_{cs_1} \mathbf{V}_1\rho_1\mathbf{V}_1\cdot\hat{\mathbf{n}}_1 dA + \int_{cs_2} \mathbf{V}_2\rho_2\mathbf{V}_2\cdot\hat{\mathbf{n}}_2 dA$$

or we can simply write it, ρ 1 V 1 A 1 into so, this V 1 will take care of the direction and here this V 2 is going to take care of this direction because this is pointing outward, if we take this direction as positive, so that is why this is negative.

So, assumption is, we have taken uniform density, we have taken uniform velocity for writing down this equation, we have taken V is always perpendicular to the area and we have taken steady state. So, V is the fluid velocity relative to the control volume.

So, m1 can be written as, $\mathbf{M}_1 = -(\rho_1 V_1 A_1)\mathbf{V}_1 = -(\rho Q)\mathbf{V}_1$. This is 1 term M 1, so, it is written as - of ρ Q into v1 and similarly, M2 here can be written as,

$$\mathbf{M}_2 = (\rho_2 V_2 A_2)\mathbf{V}_2 = (\rho Q)\mathbf{V}_2$$

. So, this term here and this term here, so, these M 1, M 2, V 1, V 2, V 2 this v 1 all our vectors.

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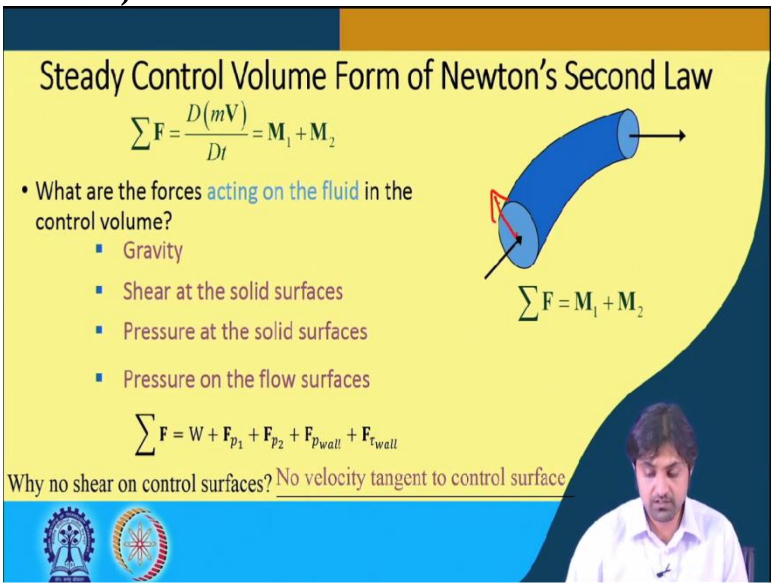

So, now, steady control volume form of Newton's second law, so, this f the net force is the sum of M 1 and M 2 as we already told you in the last slide. So, sigma x where D mV Dt is equal to M 1 + M 2. Now, the question is, what are the forces acting on fluid in the control volume. So, one is gravity, other is shear at solid surface or pressure at solid surface, pressure on the flow surfaces.

So, we can simply write, sigma F as, weight W due to gravity, pressure forces, solid surface pressure and the force on due to pressure at wall and shear at wall. This is what we have just said. So, why there is no shear on the control surface? Because there is no velocity tangent to the control surface there is no velocity that is tending to the control surface like this.

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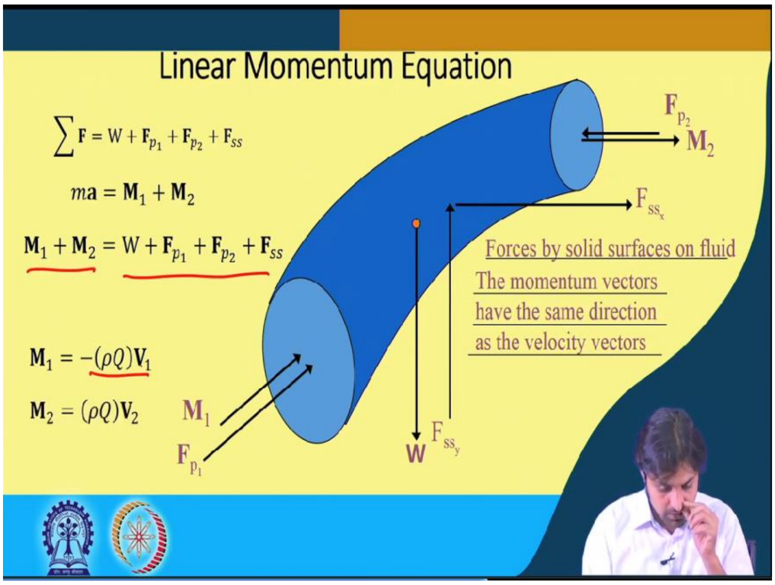

Now, going into a little zoom figure, now we have indicated what M1 is, what pressure is, what force is due to the, you know, the different pressure forces or the shear is there. This is the weight, as you can see, and this is the frictional or the shear forces F ss x and F ss y, this are the components of the shear forces in 2 different directions. So, forces by solid surface on fluid and the momentum vectors have the same direction as the velocity vectors, as we have seen, whatever the direction of the velocity is the momentum will be in the same direction. So, we can simply write F = W + F P1 + F P2 + F ss and ma force, mass into acceleration is M 1 + M 2, so, we can simply write M 1 + M 2 = W + F p1 + F p2 + F ss, and M 1 here is - ρ Q V 1 and M 2 is ρ Q V 2, plus.

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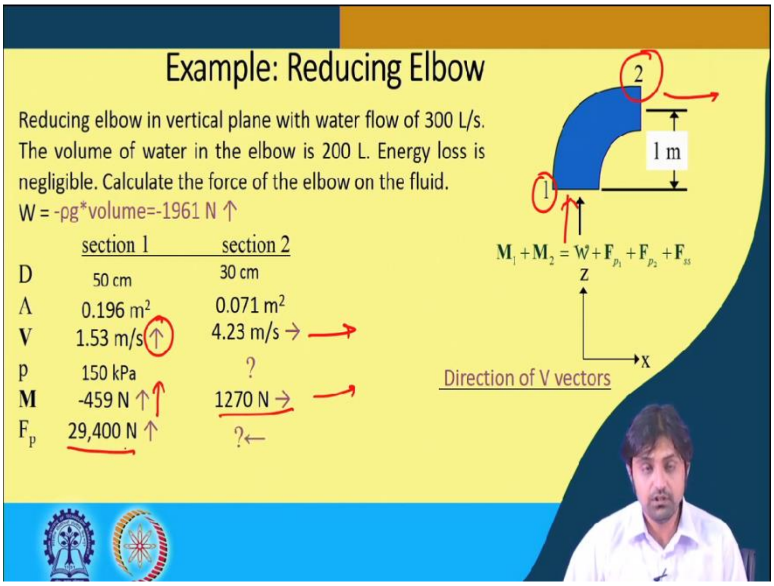

An example we are going to see is the reducing elbow using this equation, we have seen this equation already in the last slide, this is sectional 1, this is section 2, the distance between this is 1 meter. So, now question is, reducing elbow in vertical plane with water flow of 300 liters per second. So, the Q is 300 liters per second. The volume of water in the elbow is 200 litres that is the total volume. Energy loss is negligible. Calculate the forces of the elbow on the fluid.

So, this is the question, what is going to be the weight, - ρ g into volume. So, if you put volume as 200 liters. You are going to end with 1961 Newton in negative direction because weight acts downwards. Now, we treat section 1 and section 2, we have to find the diameter is 50 centimetres, this is what we have seen, and diameter at section 2 is 30 centimetres. So, area will be 0.196 meters is square in section 1 and area will be 0.071 meters square the velocity is going to be 1.53 m/s, we know the discharge and we know the area, therefore, we will know the velocity and similarly for section 2 this shows the direction so, this is the velocity directions at section 1 and this is the direction, velocity direction at this section 2 here, in this way. Now the pressure is 150 kilo Pascal. We need to find the pressure at section 2, M is because of the all the values we know we can calculate M as - 459.

How can you find M, that you can see in the slide below, - ρ Q V 1, M 2 is ρ Q v 2, you substitute that and you will be able to find M and but the direction is same as velocity vector at section 1 and it will be 1270 Newton in this direction. So, Fp is going to be 29,400 Newton, we are going to see actually a more detailed problem in the upcoming slides. But the pressure the forces due to the pressure is 29,400 Newton here. Now we have to find out the pressure force here. So that is the problem.

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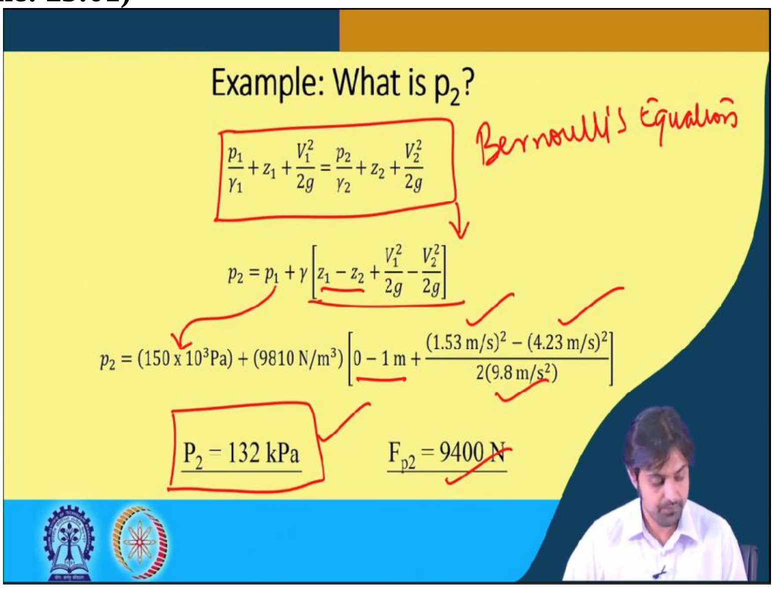

What is p 2? How do we obtain p 2? We can obtain p 2 using the Bernoulli equation. So, p 2 will be p 1 + gamma z 1, just rearrangement of this equation gives this equation and we can simply substitute the values and obtain z 1 – z 2,we already know, it is – 1 meter, V 1 square – V 2 square / 2g. So, V 1 we have calculated, V 2 we have calculated, everything is known. So, actually p2 turns out to be 132 kilopascal. Now, if we know p 2 we can calculate the pressure force that will come out to be 9400 Newton.

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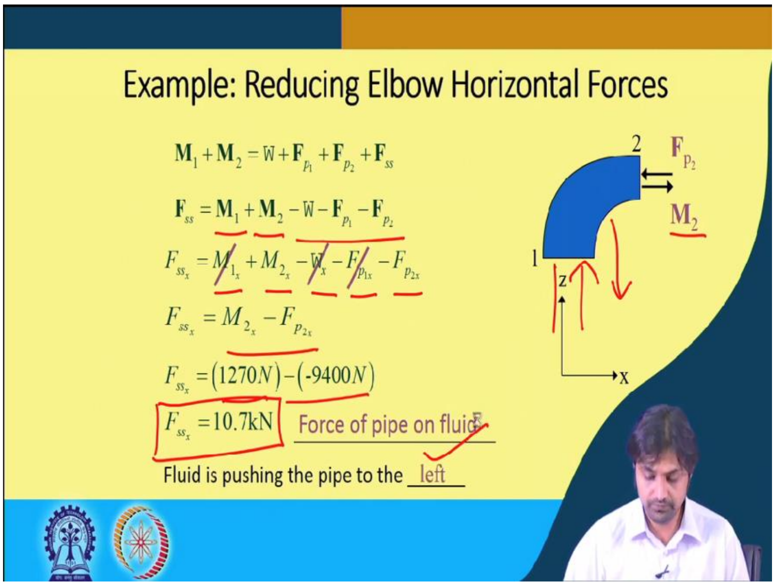

Now, it is important to find out the horizontal forces. This is what we have is, we have calculated the vertical forces. So, the same M 1 + M 2 = W + F p1 + F p2 + F ss. So, F ss is going to be M 1 + M 2 - W - F p1 - F p2 or F ss x, in x direction, because there are 2 components, it will be M1 x + M 2 x - W x - F p1 x - F p2 x.

M 1 x there is no component of 1 in horizontal direction because M 1 was in this direction, W was again vertically acting, so, this is also zero, and the pressure was also in this direction, vertical direction, therefore this is also zero. So, F ss x is M 2 x – F p 2 x, and this is M 2 x is 1270 Newton that we already calculated, - of - 9400 Newton. And this came to be 10.7 kilo Newton, very simple. Similarly, so, this is force of the pipe on fluid. So, this means that the fluid is pushing pipe to the left because F ss x is positive.

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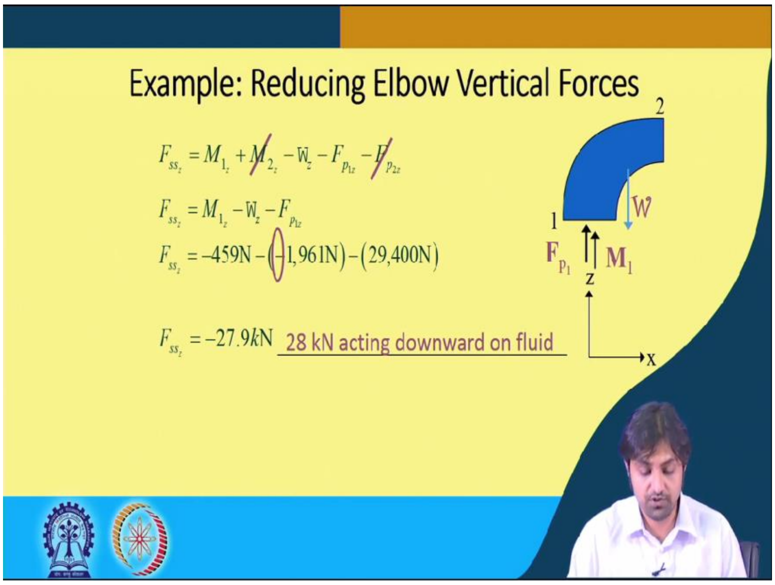

Continuing with our example, we have to find out F ss in z direction or y direction. Basically, we call this one as z. So, M 2 z because this is a horizontal, this is zero and similarly with the pressure force is it is zero. So, F ss z is M 1 - W z - Fp1 z. So, M 1z we found out it was - 459 Newton, weight we already found out and this we just we calculated last time, I mean, in the one slide before. So this minus, minus becomes positive.

But still, F ss z comes - 27.9 kilo Newton. So, this means approximately 28 kilo Newton force is acting downwards on the fluid because it is minus. Now, pipe wants to move actually up that is why this the net force is in the downward direction.

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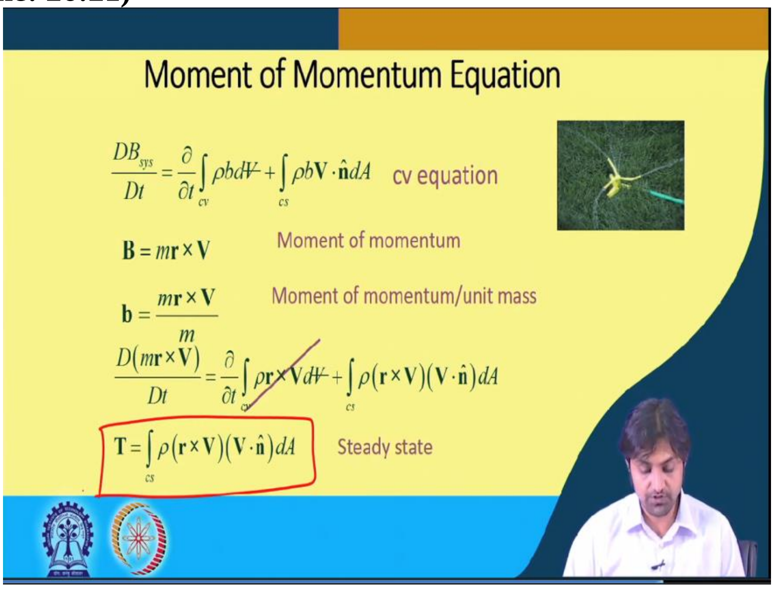

Now, similarly, there is something called moment of momentum equation, we are not going into detail, but just to show that here the B, capital B for deriving moment of momentum equation we can assume, m r into V moment of momentum or m gamma into V, this r is actually nothing but gamma and b will be m r into V / m, so, moment of momentum per unit mass. So, actually the message here is that this equation is Reynolds transport theorem can be used to derive any of these conservation principles.

So, this is going to be 0, we are not going into detail therefore this is. So, this is for steady state because this is delta t equals 0, we are not going to do the analysis.

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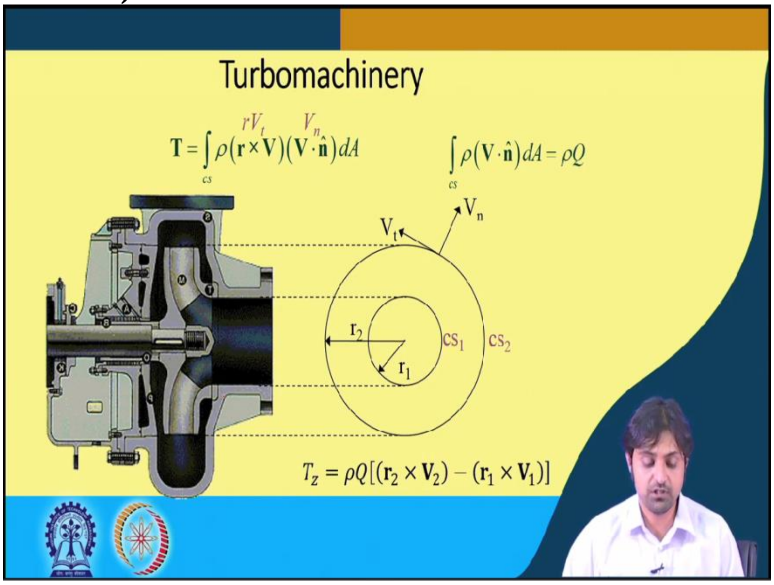

One of the one of the examples is turbo machinery. But I am going to skip this for your convenience, but to know one of the examples of moment of momentum conservation equation in real life is turbo machinery.

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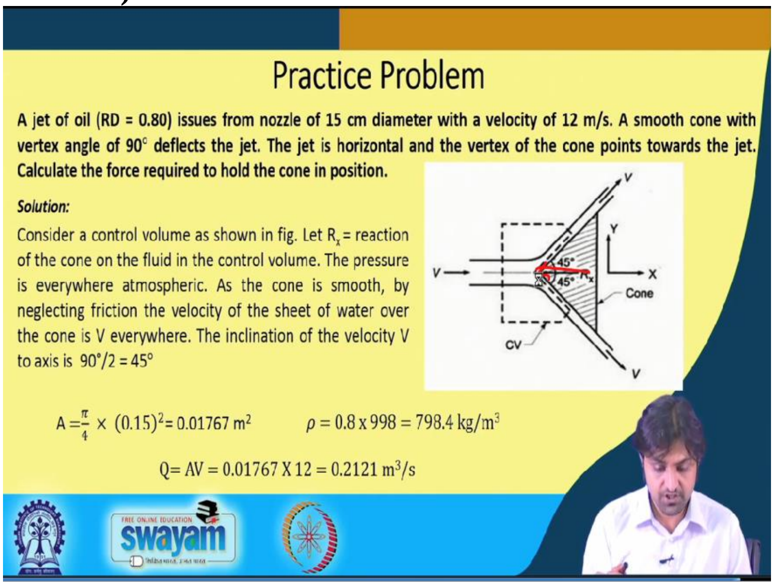

Now, we can solve 1 practice problem, this will be based mostly on the Bernoulli and momentum or mass conservation equation. So, a jet of oil relative density 0.80 issues from nozzle of 15 centimetre diameter with a velocity of 12 meters per second. A smooth cone with vertex angle of 90 degree deflects the jet. The jet is horizontal and the vertex of the cone points towards the jet. Calculate the force required to hold the cone in position.

So, we have something like this. So, this is the this is the velocity of 12 meters per second the diameter hole is 15 centimetre and this is the cone and this is 90 degrees, the jet is horizontal, the vertex of the cone points towards the jet. So, actually most important thing in this type of problems is first draw the figure and then half our problem is solved. So, we have successfully drawn the finger now. Now the solution steps is we consider the control volume as shown in the figure.

So, this is our control volume, you see, that is the first step, the pressure is everywhere atmospheric, as the cone is smooth by neglecting the frictional the friction the velocity of the sheet of water over the cone is V everywhere. The inclination of the velocity to the axis is 45 degrees that we have simply assumed. So, the area is going to be pi / 4 d square, the area here, and ρ is going to be, its oil of relative density pointed. So, ρ is going to be 0.8 into 998 a density of water, that is, 798.4 kilograms per meter cube.

So, the Q is area into velocity AV, area we have already calculated. So, area is here, and this is the velocity that we already know from here. And this comes out to be 0.2121 meter cube per second.

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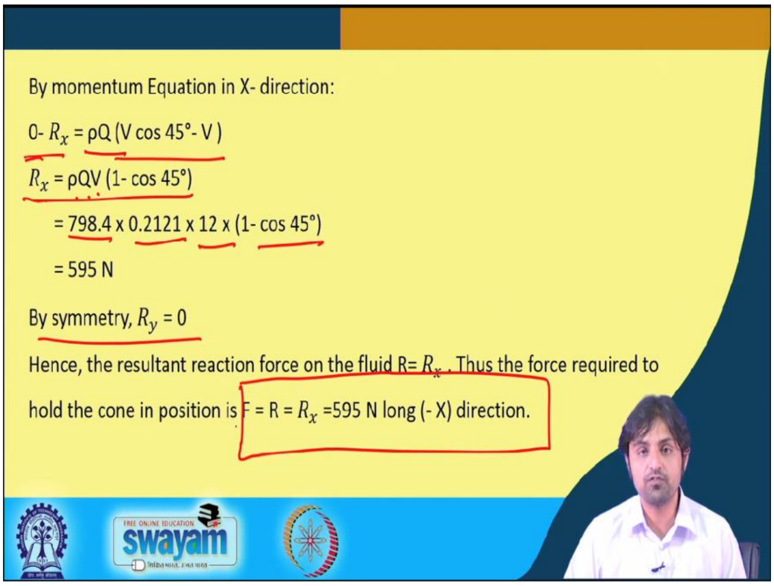

So, by momentum equation in x direction, so, you see, there is 0, if the reaction force here is R x. So, 0 - R x, is the change in momentum. And what is the change in momentum. It is ρ Q, if the velocity V was there, V cos 45 degrees - V, or V can come out, you know, R x is equal to ρ Q V 1 - cos 45 degrees. You see here, there is one force R x that is going to act, so, rate of change of momentum is ρ Q V 1 - cos 45 degrees, you substitute the value ρ QV 1 - cos 45 degrees and that is going to be R x is equal to 595 Newton. By symmetry we can see R y is equal to 0.

So, hence, the resultant reaction force on the fluid is R is equal to R x. Thus, the force required to hold the cone in position is F is equal to 595 Newton long the negative, negative x direction. This is one of the examples of this. In some point during the lecture series, we are going to solve more problems, we will have some sessions where we are going to solve only some random problems from fluid mechanics on the other topics that we are going to do.

And I think this is enough and this concludes our basics of fluid mechanics 2 section. We will start next week with laminar and turbulent flows. Thank you so much for watching.