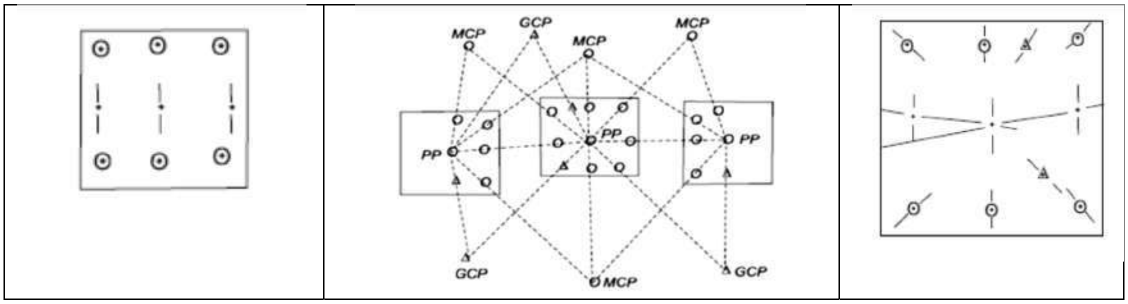

Figure 4.26 (a) Nine points on each photo, (b) Arundel method of radial line triangulation, and (c) Rays to all points from the principal point

Graphical radial triangulation is performed with simple instruments, such as mirror stereoscope, a ruler, a pencil and a tracing sheet. The steps are given below:

1.The photographs are laid out in strips and all GCPs are identified on the photographs and their numbers marked on the photographs.
2. The principal point is obtained for each photograph. The principal points are then stereoscopically transferred to the adjoining photographs as conjugate principal points.
3. Now two points, called *minor control points* (MCPs), also called *pass points* or *wing points*, are selected on both sides of the principal point of photo, at about 2 cm from the upper and lower edge of the photograph, fulfilling the following conditions:
a. The two points should be as nearly at the same elevation as the principal point.
b. The points should be at a distance from the principal point, which is equal to twice the mean base of the adjoining photographs.
c. The points should lie approximately on the bisector of the base angle on either side, and
d. The point should serve as lateral point as well.
4. The MCPs are selected and transferred stereoscopically to adjoining photographs.
5. The lateral control points (LCPs) are selected in the centre of lateral overlaps of adjacent strips to serve as connecting points between different the strips. These LCPs are selected at least at the beginning and the end of the strip and on every third photo of the strip. These are then stereoscopically transferred to photographs of adjoining strips.
6. The radial directions from the principal point to all minor control, lateral control and ground control points appearing on the photograph are drawn, through the points. A photograph on completion of the above process looks as given in Figure 4.26c.

Due to elevation differences of terrain and variations in the flying height of aircraft, the scale of photographs generally varies considerably. The photographs of a strip are required to be brought to a common scale through graphical triangulation. The plot of the strip where all the photographs, having uniform scale, are fixed in their correct relative position is called as MCP. It is preferable to start selecting MCPs somewhere in the middle of the strip to avoid accumulation of azimuthal errors.

Each strip is plotted on a transparent sheet to facilitate the drawing. The photographs are laid out in their correct relative directions so that the plotting is carried out in the right direction. With the first photograph in position below the tracing sheet, principal point base is transferred. The base line is extended up to the edge of the photograph on either side. The position of principal points of the first photograph and one of the two adjacent photographs is then transferred on the tracing sheet. The radial directions to all the points appearing on the photograph are also drawn. The first photograph is then removed and the second photograph is placed beneath the tracing sheet such that its base line and principal point coincides with the base line and principal point traced from the first photograph. The tracing sheet is firmly fixed and the principal point base to the next photograph and all the radials of the second photograph are traced out. Similarly, the third photograph is placed underneath the tracing sheet, and the above exercise is repeated. The photographs on the other side of the starting photograph are likewise completed. In this way, minor control plots of all the strips are prepared.

The MCPs of different strips are at different scales. To bring all of them to the same scale, a projection is made. Normally, in graphical method, scale of survey is nearly the same as the average scale of photographs. On the projected tracing sheet which is usually gridded, all known control data is plotted. Strips which contain two or more ground control points can be scaled down independently. If there are three or more strips, the strip which has a scale equal to average photo scale, is scaled first. Other strips are brought to this common scale through lateral control points.

The scaling is then carried out by plotting the actual distance AB between the two ground control points, between which the scaling is to be done, on a straight line drawn on a separate sheet (as shown in Figure 4.27). On this line, the distance A′B′ (from the minor control plot) is plotted with A and A′ coinciding as shown in Figure. With a pair of bow compass, a semicircle is drawn taking B′ as centre and distance BB′ as radius. The scaled position of any other point C′ on minor control plot is obtained by coinciding A′ with A and C′ falling on the line AB. Draw an arc with C′ as centre and perpendicular distance from C′ to the tangent as radius. The arc intersects the line AB at C, which is the scaled position of point C′.

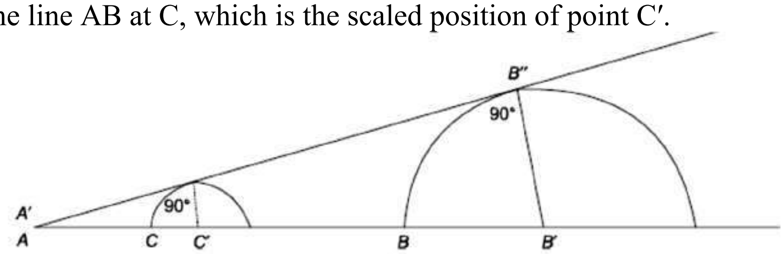


Figure 4.27 Scaling in Arundel's method

After scaling all the points of minor control plots, these points are pricked through the tracing sheet. The positions of principal points, minor control points and lateral control points of all the strips are likewise pricked on the graph sheet and adjusted so that all ground control points fall over their plotted positions and lateral control points give positions of least error. This process brings all the minor control plots at the same scale. It may be possible to place all the strips in such a way that there are no discrepancies in the final map. However, quite often, it may be necessary to fit portions of strips together using intermediate lateral tie points. It may also be necessary to rescale some strip in two or more parts if scale errors have accumulated. If the discrepancies are large, the entire process is likely to give unsatisfactory results. Additional ground control points in strips also serve as a check. The difference in position from two adjacent strips should not normally exceed 3 mm. For better accuracy, the two points used for scaling should be farthest apart.

**(b) Block Triangulation**

Block triangulation (bundles or independent models) provides the best internal strength as compared to the strip triangulation in radial triangulation method (Ackermann, 1975). The available tie points in consecutive strips assists in the roll angle recovery which is one of the weaknesses in the strip triangulation. In terms of computational aspect, the aerial triangulation methods are categorized as: analog, semi-analytical, analytical, and digital triangulation. The analytical and digital aerial triangulation methods tend to be more accurate than the analog or semi analytical aerial triangulation.

**1 Analog aerial triangulation**

This method uses a "first order" stereo-plotter to carry out the relative and approximate absolute orientation of the first model and cantilever extension. The strip or block adjustment is then performed using the resulting strip coordinates.

**2 Semi-analytical aerial triangulation**

Semi-analytical aerial triangulation or *independent model aerial triangulation* is partly analog and partly analytical procedure. Here, each stereo-pair of a strip is relatively oriented in the plotter. The coordinate system of each model being independent from the others; model coordinates of all control points and pass points have to be read and recorded for each stereo model. By means of pass points common to adjacent models, a 3D coordinate transformation is used to tie each successive model to the previous one to form a continuous strip. This strip may then be brought to ground coordinated system again through 3D coordinate transformation and adjusted numerically utilizing the polynomial equations. Relative orientation of each individual model is performed using a precision plotter.

**3. Analytical aerial triangulation**

Analytical aerial triangulation can be carried out using analytical relative orientation of each model first, and then connecting the adjacent models to form a continuous strip which is finally adjusted to ground controls. The input for analytical aerial triangulation is image coordinates measured by a comparator (in stereo-mode or mono-mode plus point transfer device). A bundle block adjustment is then performed by using all image coordinates measured in all photographs. An analytical plotter in comparator mode can also be used to measure the image coordinates.

**4. Digital aerial triangulation**

It uses a photogrammetric workstation which can display digital images. Digital aerial triangulation is similar to analytical aerial triangulation but here all the measurements are carried out utilizing the digital photographs. The procedure is fully automatic, but allows interactive guidance and interference. Adjustment can be carried out through bundle adjustment by fitting all photogrammetric measurements to ground control values in a single solution.

### 4.14.2 Orientation parameters

There are six orientation parameters, position of aerial camera and inclination of axes as (dx, dy, dz ), and ($\omega$,$\varphi$,$\kappa$). At least some control points with their known position that are visible in some photographs are required to solve these parameters. By photogrammetric methods, the coordinates can be determined only with few ground control points as contrast to ground-based triangulation methods. Using aerial resection, at least 3 control points for the calculation of 6 exterior orientation parameters of each image are required. As the process of mapping of wide areas, depending on scale, needs lots of images in different strips on a block, the number of required control points extensively increases. With these control points, an absolute orientation of the model can be carried out. In aerial triangulation method, several unknown points along with the ground control points and the coordinates of exposure stations are measured. Computation are performed to determine the coordinates of unknown points in the defined reference system. The inner orientation is performed to locate the aerial photo by using fiducial mark. Relative orientation provides a convenient means of checking most point marking and photogrammetric measurement. Pass-point and tie-points are used to connect several photos or models and strips.

### 4.14.3 Bundle adjustment

When working with software, the "bundle adjustment" algorithm will be necessary for triangulation. The name is derived from the 'bundle of light rays' that pass through each exposure station. Using algorithm, all photos are adjusted simultaneously to create an intersection of all light rays at each pass point and ground control points. This, in turn, it solves the unknown values of X, Y, and Z object space coordinates. The software also creates a 3D model of the area with lines, surfaces and textures.

A bundle of rays that originates from an object point and passes through the projective centre to the image points (Figure 4.28) forms the basic computational unit of aerial triangulation. Bundle block adjustment means the simultaneous least squares adjustment of all bundles from all the exposure stations, which implicitly includes the simultaneous recovery of the exterior orientation elements of all photographs and the positions of the object points (Faig, 1979).

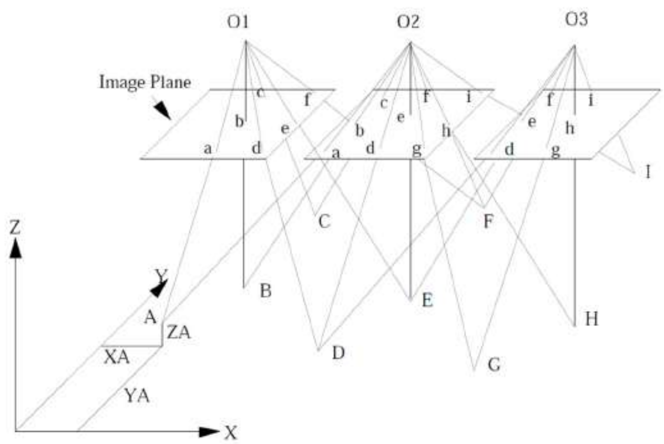


Figure 4.28 Bundle block adjustment (Faig, 1979)

The fundamental equation of aerial triangulation is the collinearity equation which states that an object point, its homologous image point and the perspective centre, are collinear (Figure 4.29). The collinearity equations are given as:

$$F_x = x_i - x_o + c\frac{m_{11}(X_i - X_O) + m_{12}(Y_i - Y_O) + m_{13}(Z_i - Z_O)}{m_{31}(X_i - X_O) + m_{32}(Y_i - Y_O) + m_{33}(Z_i - Z_O)} = 0 \qquad 2.1$$

$$F_y = y_i - y_o + c\,k_y\frac{m_{21}(X_i - X_O) + m_{22}(Y_i - Y_O) + m_{23}(Z_i - Z_O)}{m_{31}(X_i - X_O) + m_{32}(Y_i - Y_O) + m_{33}(Z_i - Z_O)} = 0 \qquad 2.2$$

where

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \qquad 2.3$$

$$\begin{bmatrix} \cos(\Phi)\cos(\kappa) & \cos(\omega)\cos(\kappa) + \sin(\omega)\sin(\Phi)\cos(\kappa) & \sin(\omega)\sin(\kappa) - \cos(\omega)\sin(\Phi)\cos(\kappa) \\ -\cos(\Phi)\sin(\kappa) & \cos(\omega)\cos(\kappa) - \sin(\omega)\sin(\Phi)\sin(\kappa) & \sin(\omega)\cos(\kappa) + \cos(\omega)\sin(\Phi)\sin(\kappa) \\ \sin(\Phi) & -\sin(\omega)\cos(\Phi) & \cos(\omega)\cos(\Phi) \end{bmatrix}$$

Where $(x_i,y_i)$ are the image coordinates, $(x_0,y_0)$ are the principal point coordinates, c is the camera constant, $m_{ij}$ is an element of the rotation matrix, $(X_i,Y_i,Z_i)$ are the object point coordinates, $(X_0,Y_0,Z_0)$ are the exposure station coordinates, M is the rotation matrix, $k_y$ is the scale factor for y-axis in digital camera (this factor is 1 for film-based camera).

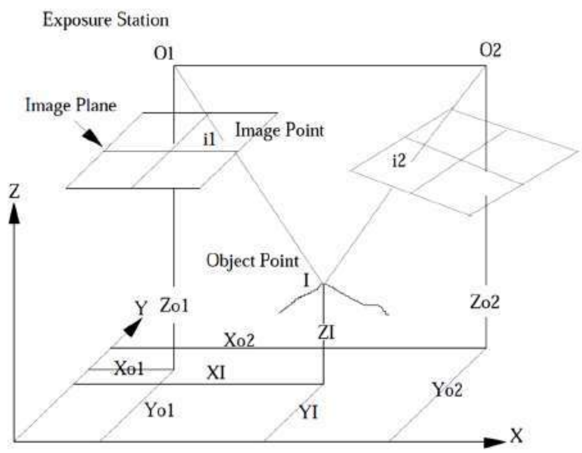


Figure 4.29 Graphical representation of collinearity condition (Su et al., 2022)

There are 6 unknowns in the collinearity equations, namely, the exterior orientation parameters $(X_0,Y_0,Z_0)$ The three rotation angles implicit in the rotation matrix M. The principal point coordinates $(x_0,y_0)$ and camera constant (c) are considered to be known for the basic bundle approach. However, this might not be true. Strong imaging geometry plus a minimum of three ground control points are needed to solve the six unknowns per bundle which are then used to determine the unknown object coordinates of other measured image points.

**4.15 Photogrammetric Mapping**

Photogrammetric mapping means evaluating and measuring the land for the determination of topography, area, contours, and location of planimetric features, by using photogrammetric methods. The ground control points required for the photogrammetric work may be established by advanced ground-based surveying methods. Photogrammetric mapping professionals utilize their knowledge to employ the appropriate methods and technologies to image, measure, calculate, reduce, and integrate the data. Photogrammetric methods are used to obtain accurate and reliable data for mapping purposes. Many times, there is a need to analyse more than one photo as the study area is large. In such cases, several images captured by aerial photography are combined first by a process called *mosaicing*. These aerial images are analysed to create topographic maps and thematic maps for various projects.

There are two ways to carry out mapping from photogrammetric techniques; manual method and digital method. In manual method, simple equipment, like a light table, magnifying lens, stereoscopes, stereo-plotters, analytical plotters, projection table, etc., may be used. Manual method is laborious, particularly when dealing with large number of photographs. The digital method of mapping would require digital photos, a workstation and a photogrammetric software. In digital methods, skilled-manpower is required to carry out the operations on digital data. The enhancement of contrast of digital images can be done easily with the software. The analysis of photos and creation of maps are much faster than the manual method. The output from manual methods is a plot, map, whereas in digital method, it is 3D model, orthophotos, DEM, digital maps, etc., which could be used further in GIS along with other maps for spatial analysis.

### 4.15.1 Mosaics

The mosaic is an assembly of overlapping aerial photographs that have been matched to form a continuous photographic representation of a portion of the Earth's surface. Photo-mosaics offer the best of both high resolution images and acquiring overlapping images over a larger area. The overlap portion allows the images to be merged to form a seamless mosaic that can be used for interpretation or map preparation. The creation of a mosaic may need less overlap than for DEM capture using photogrammetry techniques. However, larger overlaps allow the mosaic to be created from central part of the photographs which is considered to have less distortion. Figure 4.30a shows six photographs which are used to create a seamless mosaic as shown in Figure 4.30b.

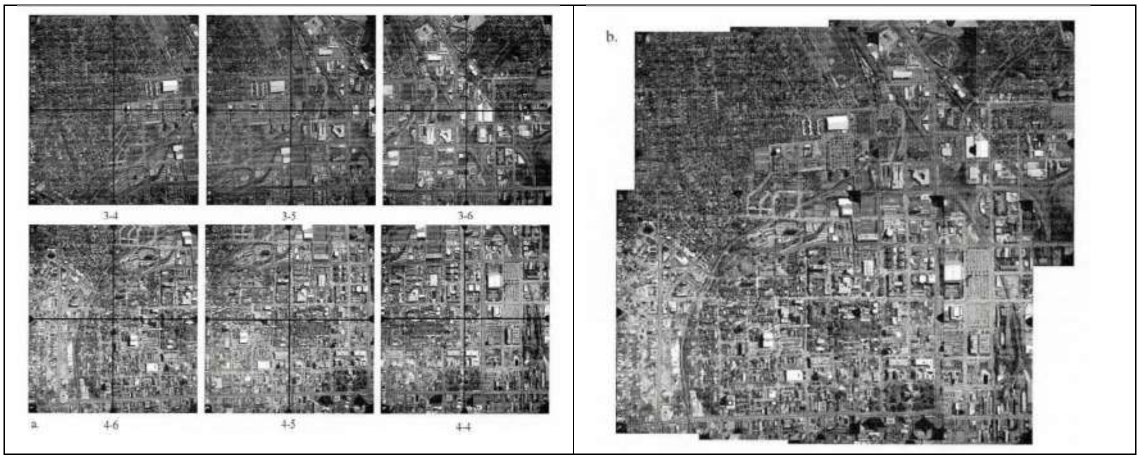


Figure 4.30 (a) six photographs, and (b) resultant mosaic (Jenson, 2013)

The steps involved in creating a mosaic are shown in Figure 4.31. Mosaics fall into two broad categories:

*1. Uncontrolled mosaic*

- Image details are matched in adjacent photos and photos are joined together manually.
- No ground controls are used.
- It is quick to prepare
- It is not as accurate as controlled mosaics but for many purposes, such as reconnaissance surveys, they are acceptable.

*2. Controlled mosaic*

- It is the most accurate.
- Photo-rectification is carried out on the images.
- Image features on adjacent photos are matched as closely as possible.
- Ground control points and corresponding points on the images are used to create the mosaic.
- Can be used for application where planimetric accuracy is important.

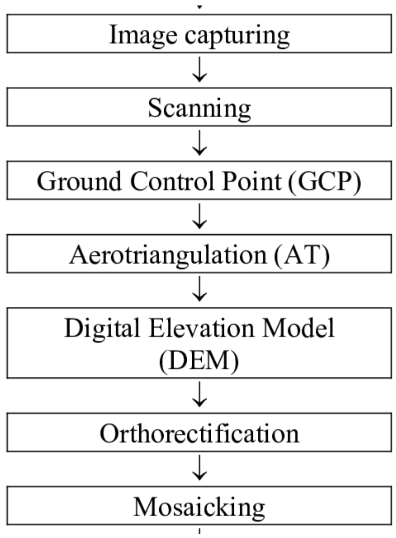

Figure 4.31 Process involved to create a mosaic (Leh et al., 2008)

Mosaics fall into two broad categories-

**(i) Photomosaics (Photomaps)**
Photomosaics are created by merging overlapping aerial photos. Additional information, such as place or road or river name can be taken form maps or ground survey. In this mosaic, there will be scale variation across the photomap, due to terrain undulations (scale of a photograph varies from point to point depending on the elevation of the point). In addition, towards the edges, there will be some relief displacement present.

**(ii) Orthophotos and Orthophoto mosaic**
An aerial photograph does not have a constant scale throughout the entire image; therefore, it can't be used directly as a map. In a photograph, except at the principal point, all other points have variation in scale depending upon the undulations in the terrain. A feature, such as a tall building, will also have shape distortion because the top of the feature will have a larger scale than the bottom of it (Figure 4.32a). An orthophoto is an aerial photograph that has been rectified so that it possesses the characteristics similar to a map. It is also known as the *map substitute*.

An orthophoto is an image that shows objects in their true positions (Figure 4.32b). They are geometrically the same as conventional maps, therefore they can also be used as basic map substitutes to take direct measurements without further adjustments. The processing to create orthophotos removes the effects of relief displacement and photographic tilt. Orthophotomaps are produced using either one or multiple overlapping orthorectified aerial photos which have the benefits of both aerial photos and maps. The rectification process is performed by combining the photogrammetric principles with the DEM data (Wolf and Dewitt, 2000).

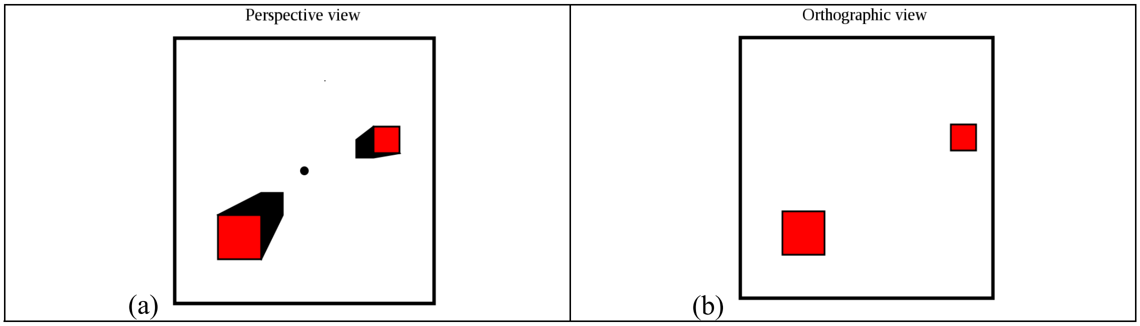


Figure 4.32 Buildings on (a) aerial photograph, and (b) orthophoto

Map revision or map updation becomes easy using orthophotos. Orthophotos have been used for many years for a diverse range of applications, including GIS. The disadvantage of orthophoto is that the elevations of surface (topography) and also the height of every feature (buildings, trees etc.) above that surface should be known to create an accurate DEM, otherwise there will be some error. In past, orthophotos were produced with a photogrammetric stereo-plotter, but today with the advent of digital photogrammetric methods, orthophotos can be produced on a PC using the appropriate photogrammetric software. An orthophoto is produced by computing the scale and position distortions of each pixel of aerial photograph, re-scaling and re-positioning the pixels in a new generated image.

**4.15.2 Stereo-plotting instruments**

Stereoscopic plotting instruments or stereo-plotters are designed to provide accurate solutions for object point positions from their corresponding image positions in a stereo-pair. They are capable of producing accurate 3-D coordinates of x, y, and z object space coordinates after proper orientation and calibration. The modern stereoplotters can also handle oblique or terrestrial photos. The primary uses of stereo-plotters are to create topographic maps and digital files of topographic information.

An overlapping pair of aerial photos or transparencies or diapositives, are placed in two stereo-plotter projectors, as shown in Figure 4.33. This process is called *interior orientation*. With the diapositives in place, light rays are projected through them; and when rays from corresponding images on the left and right diapositives intersect below, they create a stereo-model. In creating the intersections of corresponding light rays, two projectors are oriented so that the diapositives have the exact relative angular orientation to each other in the projectors, similar to the negative in the camera at the time of exposure. The process is called *relative orientation*, which creates a true 3-D stereo-model of the overlap region. After relative orientation is completed, *absolute orientation* is performed. In this process, the stereo-model is brought to the desired scale and levelled with respect to a reference datum.

After completing the orientations, measurements from the 3-D model may be made and recorded, but it is now done in digital form. The position of any point/object is determined by using a reference mark (called the *floating mark*) in contact with the 3-D model point. At the position of the reference mark, the 3-D coordinates ($x$, $y$, and $z$) are obtained through either an analogue or a digital solution. Planimetric ($x$, $y$) positions and elevations ($z$) of various points/objects can thus be obtained.

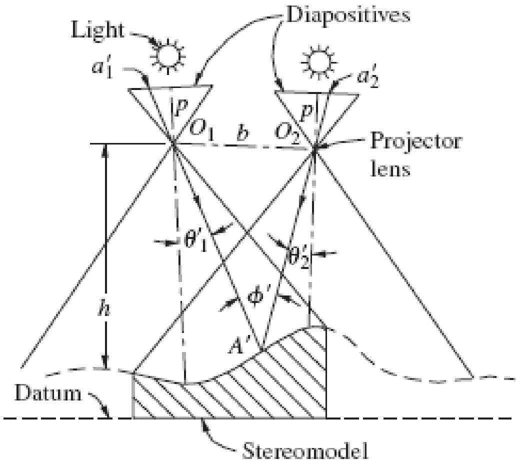

Figure 4.33 Creating a stereo-model in stereo-plotters (ASPRS, 1980)

### 4.15.3 Types of stereo-plotting instruments

Several stereoscopic plotting instruments have been developed over the past; each with different features. The stereo-plotters can be classified into in to four groups: (1) *direct optical projection* instruments, (2) instruments with *mechanical or optical-mechanical projection,* (3) *analytical* stereo-plotters, and (4) *softcopy* stereo-plotters. The first-generation stereo-plotters were of direct optical projection design, creating a 3-D stereo-model by projecting the transparency images through projector lenses, as illustrated in Figure 4.33. The model is formed by the intersections of light rays from corresponding images of the left and right diapositives. An operator is able to view the model directly, and make measurements on it by intercepting projected rays on a viewing screen.

Instruments based on mechanical projection or optical-mechanical projection create a 3-D model from which measurements are taken. Their method of projection, however, is a simulation of direct projection of light rays by mechanical or optical-mechanical means. An operator views the diapositives stereoscopically directly through a binocular train and carries out measurements. Figure 4.34 shows an optical-mechanical projection instrument.

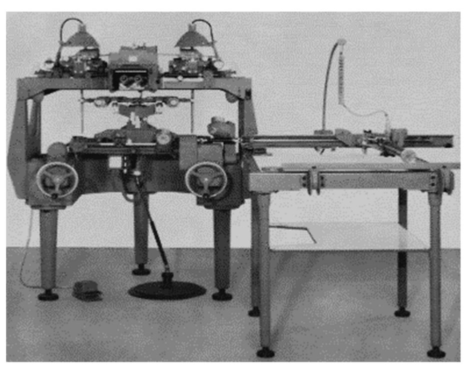
Figure 4.34 An optical-mechanical projection instrument (Kellie and Valentine, 1987)

Photogrammetric operations can also be performed by mathematical modelling. This method is called *analytical, numerical* or *computational photogrammetry*. With development in optics and mechanics, the analogue photogrammetric instruments have improved to attain high accuracy. With the evolution of computers, analogue instruments have been replaced by analytical plotters, where single or a pair of photographs is placed which digitally records image coordinates (using mono or stereo-comparators). Analytical stereo-plotters form a stereo-model using a mathematical procedure through a computer, as shown in Figure 4.35. As with mechanical plotters, an operator views the diapositives stereoscopically directly through a binocular train. The movements of the stereoscopic images are introduced by servomotors which are under computer control. The procedure to process photos in analytical plotters is given in Figure 4.36.

Figure 4.35 An analytical plotter (Egles and Kasser, 2001)

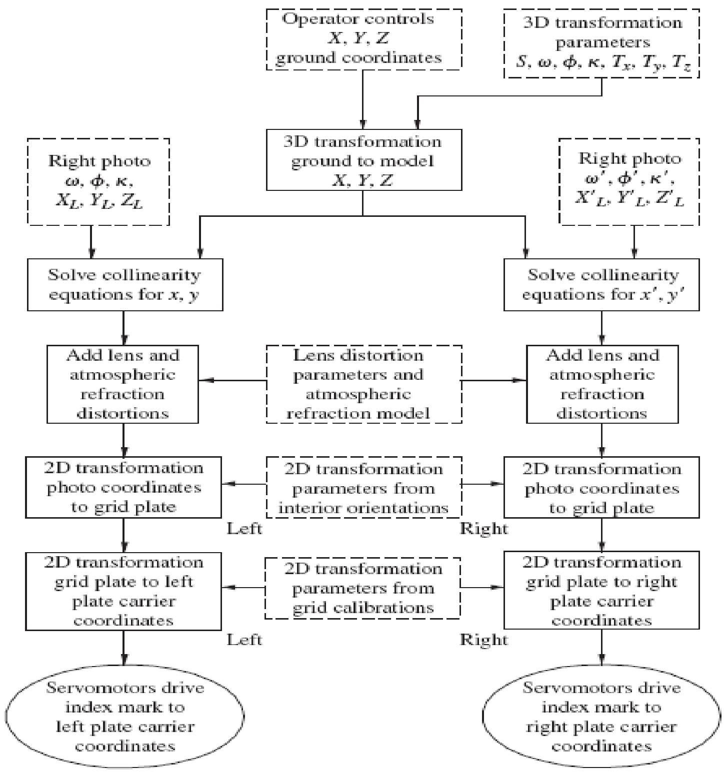


Figure 4.36 Procedure used in analytical photogrammetry (Grave, 1996)

Softcopy stereo-plotters are the most recent developments. These plotters operate in the same manner as analytical stereo-plotters, except that instead of viewing film (hard-copy) diapositives through binocular optics, softcopy photographs are displayed on a computer screen. Special viewing system enables the operator to view the left image with the left eye and the right image with the right eye in order to see in stereo.

In digital or soft copy photogrammetry, digital images are used with wide range of data processing operations in Digital Photogrammetric Workstation (DPW). A DPW is shown in Figure 4.37. The developments in digital photogrammetry have resulted in DPW and specialised software. Some of the well-known digital photogrammetric systems are LH Systems, Z/I Imaging, and ERDAS. The DPWs are more user-friendly than the analytical plotters, and have image processing capabilities, such as enlargements, reductions, contrast enhancements, etc. The DPW is more reliable and accurate, since no calibration is required (Manugula et al., 2018). Flow diagram of the processes involved is shown in Figure 4.38. The DPWs have the potential to automate photogrammetric applications, such as aerial triangulation, DEM generation, and orthophoto production. As more photogrammetric procedures will be automated, the operation of a DPW would require less specialized skilled manpower (Egles and Kasser, 2001).

Figure 4.37 A digital workstation supported with photogrammetric software (Eglas and Kasser, 2001)

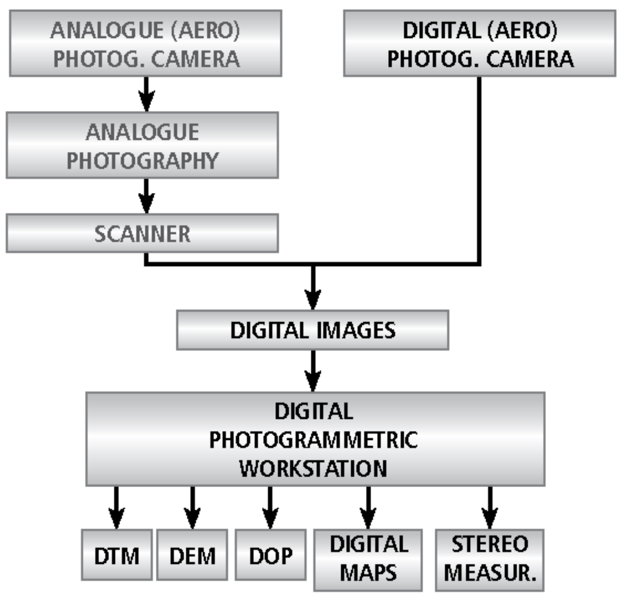


Figure 4.38 Processing of photos in a DPW (Balenovie et al., 2011)

### 4.15.4 Photgrammetric software

There are several specialized software for photogrammetry, which could be used to analyze the 2D images captured by photogrammetric camera. Mapping or the surface creation process is also done with these photogrammetry software. The software can be categorized into two classes; (a) Free source, and (b) Commercial software. A comparison of some of the photogrammetry software is presented in Table 4.2.

Table 4.2 Comparison of some photogrammetric software (Formlabs, 2021)

| Quality | Speed | Features | User-friendliness |
|---|---|---|---|
| 3DF Zephyr | ★★★★☆ | ★★★☆☆ | ★★★★★ |

| | | | |
|---|---|---|---|
| Agisoft Metashape | ★★★★☆ | ★★☆☆☆ | ★★★★☆ |
| Autodesk ReCap | ★★★★★ | ★★★☆☆ | ★★★☆☆ |
| COLMAP | ★★★★☆ | ★★★★☆ | ★★☆☆☆ |
| iWitness | ★★★★★ | ★★☆☆☆ | ★★★☆☆ |
| Meshroom | ★★★★☆ | ★★☆☆☆ | ★★☆☆☆ |
| RealityCapture | ★★★★★ | ★★★★★ | ★★★★☆ |
| Regard3D | ★★★☆☆ | ★★☆☆☆ | ★★☆☆☆ |
| VisualSFM | ★★★☆☆ | ★★★☆☆ | ★★☆☆☆ |

**(a) Free Source Software**

**(i) MicMac:** MicMac is useful for projects involving environmental protection, cultural heritage imaging and preservation or forestry. It can use from close-range to aerial images.

**(ii) Meshroom:** Meshroom is a photogrammetric computer vision framework. This 3D reconstruction software is easy to use. It can create textured mesh automatically using a node-based workflow.

**(iii) 3DF Zephyr Free:** It is the free version of the software *3DF Zephyr*; a complete and efficient software for photogrammetry. It is a good software for the beginners for 3D processing. It offers 3D reconstruction tools and basic editing tools.

**(iv) Visual SFM:** Visual SFM is a 3D reconstruction tool, using Structure from Motion (SfM). This GUI application is an easy photogrammetry software to use; matching and making the automatic reconstruction. It is a simple software tool with an automatic process.

**(v) Colmap:** Colmap is a general-purpose SfM software. It has graphical user interface (GUI), and offers all the basic tools needed to create a 3D model using one or several photographs.

**(vi) Regard3D:** Regard3D is a SfM program, allowing to generate 3D models from a series of photographs. This program is offering powerful tools with tutorials available on the website to get started.

**(b) Commercial Software**

**(i) ContextCapture:** Previously *Acute 3D*, it is used to create a finished 3D model with photographs without any human intervention. This process is easier than 3D scanning and more precise than 3D modelling. This photogrammetry solution allows working on large infrastructure projects, such as cityscapes. This program enables users to edit 3D meshes, generate cross-sections, and extract ground and break-lines.

**(ii) Reality Capture:** Reality Capture is a complete photogrammetry solution, and claims to be 10 times faster than any other photogrammetry solution. This program is able to calculate meshes and textures, and allows working with a lot of different file formats.

**(iii) 3DF ZEPHYR:** This software allows reconstructing a 3D digital representation with images, automatically. This software has a basic and free version. The advanced versions allow to get laser-scanned objects, or to work more precisely with aerial photographs.

**(iv) Autodesk ReCap Pro:** Autodesk ReCap (previously known as *Autodesk ReMake*) is a software allowing to create accurate 3D models using reality captures. It can also measure and edit point cloud data. It has a wide range of tools, for example, to clean the unwanted objects to work more specifically on a precise object.

**(v) Trimble Inpho:** Trimble Inpho is a photogrammetry software dedicated to geospatial use. It can transform aerial images into precise points cloud and surface models. It is a complete software with a wide range of modules.

**(vi) iWitnessPRO:** iWitnessPRO is a professional photogrammetry software, supporting both close-range photogrammetry and aerial photogrammetry. It is easy to use, supports GCPs and allows to generate orthoimages and digital surface models (DSMs).

**(vii) PhotoModeler Pro 5:** PhotoModeler Pro 5 is a plugin that is available with the 3D modeling software Rhino. It is able to create realistic and accurate models from images. The PhotoModeler photogrammetry software can also extract measurements and models from images taken with ordinary cameras.

**(viii) Correlator 3-D:** Correlator3D is a mapping software developed for terrain modeling and aerial photogrammetry. This software can produce dense Digital Surface Models (DSM), Digital Terrain Models (DTM), dense point clouds, ortho-mosaics and vectorised 3-D features.

**Unit Summary**

This unit covers various aspects of photogrammetry. Various types of photographs and associated technical term used in photogrammetry are defined. Concept of scale and relief displacement is explained. Relationships have been established to determine the scale of the photograph as well as reliedf displacement. The principle of stereoscopy and use of stereoscope are covered. Use of parallax bar to determine the elevations of various points has been discussed. Aerial triangulation plays an important role for providing horizontal as well as vertical controls from several aerial photographs; and therefore it has been described. The use of digital aerial photographs which can be employed for large number of applications, including creation of DEM, DSM etc. At the end, some photogrammetric software are also given.

**Solved Examples**

**Example 4.1:**

A vertical photograph was taken at an altitude of 1200 m above *msl*. Find out the scale of photograph for a terrain lying at an average elevation of (i) 80 m and (ii) 300 m, if the foal length of the camera is 15 cm.

**Solution:**

H = 1200 m, f = 15 cm, and (i) h = 80 m, (ii) h = 300 m

The scale of a photograph is given by = f / (H- h)

(i) Scale = 15 / (1200 - 80 ) = 1 / 7467. It means 1 cm on the photo is equal to 7467 cm on the ground.

(ii) Scale = 15 / (1200 - 300) = 1 / 6000

**Example 4.2:**

A camera of 152 mm focal length lens is used to take a vertical photograph from a flying height of 2780 m above mean sea level. If the terrain is flat having an elevation of 500 m above *msl*, determine the scale of the photograph.

**Solution:**
f = 152 mm, H = 2780 m, h = 500 m
S = f / (H – h)
= (152 / 1000) / (2780 – 500)
= 0.152 / 2280 = 1 / 15000
The scale of photograph is 1 : 15000

**Example 4.3:**
A camera of focal length 20 cm is used to take vertical photographs of a terrain having an average elevation of 1600 m above *msl*. At what height above *msl* an aircraft must fly to take the photographs at 1:10,000 scale.

**Solution:**
f = 20 cm, h = 1600 m, and scale = 1:10,000
S = f / (H- h)
H – h = S f or H = S f + h
H = (20/100) (10,000) + 1600 m
= 3600 m

**Example 4.4:**
A line AB 300 m long, lying at an elevation of 600 m above *msl,* measures 9 cm on a vertical photograph. The focal length of the camera lens is 30 cm. Determine the average scale of the photograph if the average elevation of terrain is 700 m above *msl*.

**Solution:**
Ground distance = 300 m, map distance = 9 cm, f = 30 cm, h = 600 m
Map distance / Ground distance = S = f / (H-h)
0.09 / 300 = 0.30 / (H - 600)
H - 600 = (0.30 * 300) / 0.09
H = 1000 + 600 = 1600 m
Now
$S_{av} = f / (H – h_{av})$
$S_{av} = 0.30 / (1600 - 700)$
= 1 / 3000

**Example 4.5:**
A line AB appears measures 12.70 cm on an aerial photograph and the corresponding line measures 2.54 cm on a map at a scale 1/50,000. The terrain has an average elevation of 200 m above *msl*. Calculate the flying height of the aircraft, above *msl*, if the photograph was taken with camera of 16 cm focal length.

**Solution:**
Photo distance = 12.70 cm, map distance = 2.54 cm, map scale = 1 / 50,000, $h_{av}$ = 200 m, f = 16 cm
Photo distance / map distance = Photo scale / map scale
12.70 / 2.54 = Photo scale / (1 / 50000)

5.0 = 50000 * Photo scale
Photo scale = 1 / 10,000
Now S = f / (H - h)
1/ 10,000 = 0.16 / (H – 200)
(H - 200) = 0.16 * 10000
H = 1600 + 200
H = 1800 m

**Example 4.6:**
A square flat area on the ground with side as 100 m and uniform height, appears as 1 $cm^2$ on a vertical aerial photograph. The topographic map shows that a contour of 650 m passes through the area. If focal length of camera lens is 150 mm, determine the flying height of the aircraft.

**Solution:**
1 $cm^2$ = 100 $m^2$
S = 1 cm = 100 m
1 cm = 100 × 100 cm
Scale = 1/10,000
h = 650 m,  f = 150 mm = 0.15 m
S = f / (H – h)
1 / 10,000 = 0.15 / (H – 650)
H = (0.15 * 10000) + 650
= 1500 + 650 = 2150 m

**Example 4.7:**
In aerial mapping, a camera with a 100 cm focal length lens was used. Determine the height at which the airplane must fly, so that a 1 km long road on the ground fits exactly on the 20 cm photographic format of the camera.

**Solution:**
f = 100 cm = 1 m, size of object, d1= 1 km =1000 m, size of image, d2 = 20 cm = 0.2 m
Scale = 0.2 / 1000 = f / H
H = (1000 * 1) / 0.2
H = 5000 m 0r 5 km

**Example 4.8:**
A rectangular area 130 kmx120 km is to be mapped from aerial photographs taken to a scale of 1/20000. The focal length of the lens of the camera to be used is 152 mm and each print is to be 230 mm square. Provision is to be made for a 60% overlap between successive exposures and a 25% lateral overlap. Find (a) the average height above ground at which the aircraft must operate; (b) the time interval between exposures in any one strip if the operating speed of the aircraft is 200 km/h; and (c) the minimum number of photographs required.

**Solution:**
a) Average height above ground level:

$$S_{av} = \frac{f}{H - h_{av}}, \qquad but\, h_{av} = 0\ at\ ground\ level$$

$$Hence, H = \frac{f}{S_{av}} = \frac{152mm}{1/20000} = 152 * 20000 * 10^{-3} = 3040m$$

(b) Let the flight line be parallel to the 130 km length. Since there is 60% overlap between successive exposures, the effective length of each photograph is 40% of 230 mm, i.e., 0.4*230 = 92 mm

The ground distance covered by this photo length is

$92 \text{ mm} * 20000 * 10^{-3} = 1840 \text{ m}$

Number of photograph per strip= $\frac{130,000}{1840} = 70.65 \cong 71 \; photos$

The operating speed of the aircraft is 200 km/h. To cover a length of 130 km, the aircraft needs 130 / 200 = 0.65 hour.

Since the exposures are at regular intervals

Time interval between exposures= $\frac{0.65}{70.65} \; hour = 33.12 \sec$

(c) The width of the area to be photographed is 120 km. A 25% lateral overlap results in an effective photo length of 230 mm is 0.75 * 230 =172.5 mm. The ground distance covered by this width is $172.5 * 20000 * 10^{-3} = 3450$ m.

Number of strips = $\frac{120,000}{3,450} \cong 35 strips$

Minimum number of photograph required = 71*35=2485

**Example 4.9:**

Two points A and B of elevations of 400 m and 200 m, respectively above *msl*, appear on a vertical photograph having focal length of 20 cm and taken with flying height of 2000 m above *msl*. The photographic co-ordinates of A and B are respectively (2.75, 1.39) cm, and (-1.80, 3.72) cm. Determine the length of line AB on the ground.

**Solution:**

The ground co-ordinates (X and Y) are given by

$X = x (H - h) / f$

and $Y = y (H - h) / f$

where x and y are photo coordinates.

$X_A = (2.75 / 100) * (2000 - 400) / 0.20 = 220$ m

$Y_A = (1.39 / 100) * (2000 - 400) / 0.20 = 111.2$ m

$X_B = (-1.80 / 100) * (2000 - 400) / 0.20 = -162$ m

$Y_B = (3.72 / 100) * (2000 - 400) / 0.20 = 334.8$ m

$AB = \sqrt{[(X_A - X_B)^2 + (Y_A - Y_B)^2]}$

$AB = \sqrt{[\{220 - (-162)\}^2 + (111.2 - 334.8)^2]}$

$AB = 10^2 \sqrt{[(15.592)^2 + (5.0)^2]}$

AB = 442.87 m

**Example 4.10:**

An area of 20 km x 16 km was to be covered by vertical aerial photograph at 1: 15,000 scale with the longitudinal overlap as 60% and the side overlap as 30% scale. Determine the number of photographs (take 23 cm x 23 cm as standard format) required to cover the entire area.

**Solution:**