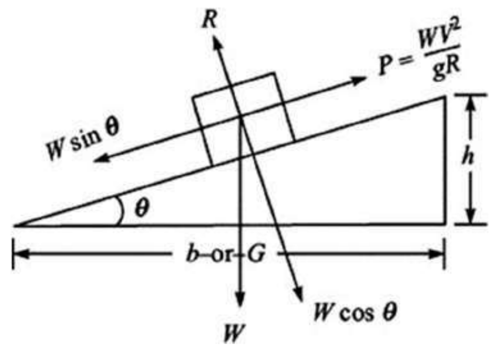


Figure 2.17 Depiction of super-elevation

If W is the weight of vehicle acting vertically downwards, P is the centrifugal force acting horizontally, v is the speed of vehicle in meters/sec, g is the acceleration due to gravity, 9.81 meters/sec^2, R is the radius of the curve in meters, h is the super-elevation in meters, b or G is the breadth of the road or the distance between the centres of the rails in meters, then-

For maintaining the equilibrium, resultant of the weight and the centrifugal force should be equal and opposite to the reaction perpendicular to the road or rail surface.

The centrifugal force can be represented as, $P = \frac{Wv^2}{gR}$

So
$$W \sin\theta = \frac{Wv^2}{gR}$$
$$\sin\theta = \frac{v^2}{gR}$$

If θ is the inclination of the road or rail surface from ground which is a very small angle, we can write;

$$\sin\theta = \tan\theta = \frac{v^2}{gR}$$

Super-elevation $h = b \tan\theta$

$$h = \frac{bv^2}{gR} \tag{2.50}$$

2.6.2 Length of a transition curve

The length of the transition curve may be computed from (i) arbitrary gradient, (ii) the time rate, and (iii) rate of change of radial acceleration. It can be determined in the following ways;

1. The length may be already assumed on the basis of experience and judgement as say, 100 m.
2. The length may be such that the super-elevation is applied at a uniform rate of 1 in 300 to 1 in 1200. If h is the amount of super-elevation, then the length of the transition in curve may be from 300h to 1200h.
3. The length of the transition curve may be such that the super-elevation is applied at an arbitrary time rate of 'a' cm/sec. The value of 'a' usually varies from 2.5 cm to 5 cm.
4. The length of the transition curve may be such that rate of change of radial acceleration does not exceed a certain value, which is generally 30 cm/sec^2. It would help in smooth moving of the vehicle and the passengers when moving over the curve.

If L is the length of the transition curve in meters, v is the speed in meters/sec, h is the amount of super-elevation in centimeters, *1 in n* is the rate at which super-elevation is provided, a is the time rate, R is the radius of the curve in meters, and C is the rate of change of radial acceleration in meters/sec^2. then-

$$L = n\,h$$
$$L = n\,(bv^2 / gR)$$
$$L = nbv^2 / gR \qquad (2.51)$$

Other approach could be if the time taken by a vehicle in passing over the length of the transition curve is $\frac{L}{v}$ sec., the super-elevation (h) attained in this time is-

$$h = \frac{L}{v} \times a$$
$$L = \frac{hv}{a} \qquad (2.52)$$

The third approach is to compute radial acceleration.

The radial acceleration on the circular curve $= \frac{v^2}{R}$

Time taken by a vehicle to pass over the transition curve $= \frac{L}{v}$ sec.

Radial acceleration attained in this time $= C\frac{L}{v}\,m/\sec^2$

so $\frac{v^2}{R} = C\frac{L}{v}$

or $L = \frac{v^3}{CR}$ (2.53)

The length of transition curve can be determined using above relationships (2.51 to 2.53).

2.6.3 Characteristics of a transition curve

In Figure 2.18, a circular curve EE′ of radius R, and two transition curves T_1E and $E'T_1$ at the two ends, have been inserted between the two straights. The two straights AB and BC make a deflection angle Δ. It is clear from the figure that in order to fit in the transition curves at the ends, a circular imaginary curve ($T_1F_1T_2$) of slightly greater radius has to be shifted towards the centre as ($E_1EFE'E_1$). The distance through which the curve is shifted is known as shift (S) of the curve, and is equal to $\frac{L^2}{24R}$, where L is the length of each transition curve and R is the radius of the desired circular curve (EFE′). The length of shift (T_1E_1) and the transition curve (TE) mutually bisect each other.

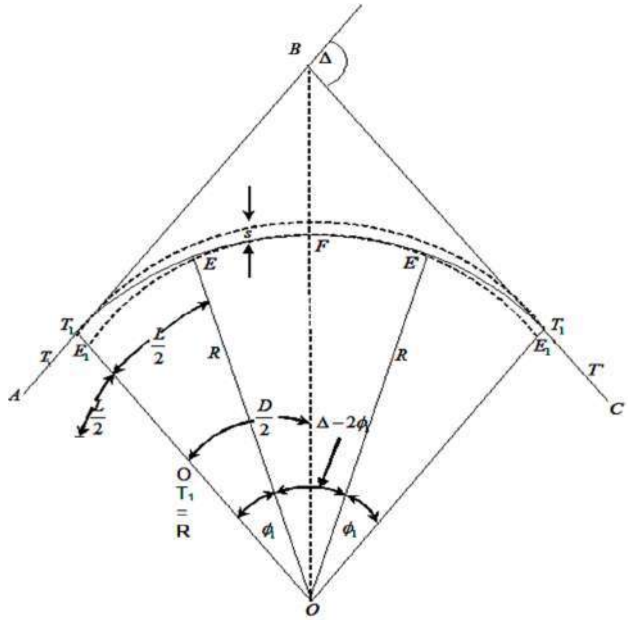


Figure 2.18

The tangent length for the combined curve is-

$$BT = BT_1 + T_1 = OT_1 \tan\frac{\Delta}{2} + \frac{L}{2}$$

$$= (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}$$

The spiral angle $\phi_1 = \frac{L/2}{R} = \frac{L}{2R}$ radians (2.54)

The central angle for the circular curve, $\angle EOE' = \Delta - 2\phi_1$

Length of the circular curve EFE' $= \frac{\pi R(\Delta - 2\phi_1)}{180^0}$

where Δ and ϕ_1 angles are in degrees

Length of the combined curve TEE'T' $= TE + EE' + E'T'$

$$= L + \frac{\pi R(\Delta - 2\phi_1)}{180^0} + L$$

$$= \frac{\pi R(\Delta - 2\phi_1)}{180^0} + 2L \quad (2.55)$$

Chainage of beginning (T) of the combined curve = chainage of the intersection point (B) - total tangent length for the combined curve (BT).

Chainage of the junction point (E) of the transition curve and the circular curve = chainage of T + length of the transition curve (L).

Chainage of the other junction point (E′) of the circular curve and the other transition curve = chainage of E + length of the circular curve.

Chainage of the end point (T′) of the combined curve = chainage of E′ + length of the transition curve.

Check: The chainage of T′ thus obtained should be = chainage of T + length of the combined curve.

Note: The points on the combined curve should be marked using chainages so that there will be sub-chords at each end of the transition curves and of the circular curve.

The deflection angle for any point on the transition curve distant *l* from the beginning of the combined curve (T),

$$\alpha = \frac{l^2}{6RL} radians = \frac{1800 l^2}{\pi RL} minutes$$

$$= \frac{573 l^2}{RL} minutes \qquad (2.56)$$

Check: The deflection angle for full length of the transition curve is-

$$\alpha = \frac{l^2}{6RL} = \frac{L^2}{6RL} \qquad (l = L)$$

$$= \frac{L}{6R} radians = \frac{1}{3}\phi_1 \quad \text{(using equation 2.54)} \qquad (2.57)$$

The deflection angles for the circular curve are found from, $\delta_n = 1718.9 \frac{C_n}{R} minutes$.

Check: The deflection angle for full length of the circular curve, $\Delta_n = \frac{1}{2} \times central \ \ angle$

i.e., $\Delta_n = \frac{1}{2}(\Delta - 2\phi_1)$

The offsets for the transition curve are found from perpendicular offset-

$y = \frac{x^3}{6RL}$ where x is measured along the tangent TB.
(2.58)

Tangent offset, $y = \frac{l^3}{6RL}$ where *l* is distance measured along the curve. (2.59)

Checks:

(a) The offset at half the length of the transition curve is-

$$y = \frac{l^3}{6RL} = \frac{(L/2)^3}{6RL} \qquad (l = L/2)$$

$$= \frac{L^2}{48R} \qquad (2.60)$$

(b) The offset at junction point on the transition curve is-

$$y = \frac{L^3}{6RL} = \frac{L^2}{6R} \text{(here } I = L) \qquad (2.61)$$

The offsets for the circular curve from chords produced are found from equation 2.20.

$$O_n = \frac{C_n(C_{n-1} + C_n)}{2R}$$

2.7 Vertical Curves

Vertical curves are introduced at changes of gradient to maintain the good visibility as well as avoid any impact while the vehicle is moving along the curve. These are provided to secure safety, appearance and visibility. These curves are set out in a vertical plane to obtain a gradual change of gradient, either higher to lower or lower to higher. These curves may be circular or parabolic, but the latter shape is commonly used. The most common practice has been to use parabolic curves in summit curves, mainly because of the ease of setting them out on the field and the comfortable transition from one gradient to another to provide excellent riding comfort. In case of valley curves, use of cubic parabola is preferred as it closely approximates the ideal transition requirements. Although, for small gradient angles, the difference between a circular and a parabolic curve is negligible.

Designing a vertical curve consists principally of deciding on the proper length of the curve. The longer a curve is, the more gradual the transition will be from one grade to the next; the shorter the curve, the more abrupt the change will be. The change must be gradual enough to provide the required sight distance (Figure 2.19). The sight distance depends on the speed for which the road is designed, the passing or non-passing distance requirements, driver's reaction time, braking time, stopping distance, eye level, and the height of objects. A typical eye level used for designs is between 3.75-4.5 feet; typical object heights are 4 inches to 1.5 feet. For a valley curve, the sight distance (SD) will usually not be significant during daylight, but the nighttime sight distance must be considered when the reach of headlights may be limited by the abruptness of the curve.

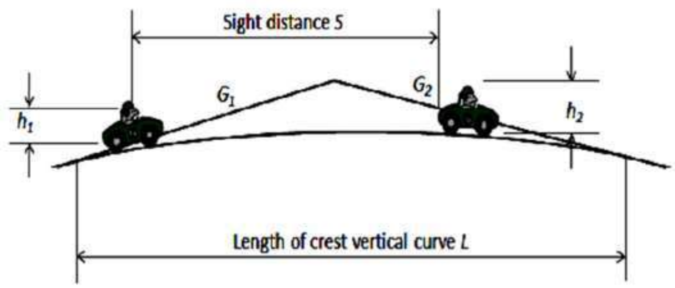


Figure 2.19 Sight distance in vertical summit curve

The gradient is expressed in two ways:
(a) As a percentage, e.g., 1%, 1.5%, etc., and
(b) As a ratio, like 1 in n, where n is the horizontal distance and 1 represents vertical distance, e.g., 1 in 100, 1 in 200, etc.

The gradient may be 'rise' or 'fall'. An up gradient is known as 'rise' and is denoted by a positive sign, and a down gradient is known as 'fall' and is indicated by a negative sign. In upgrade curve, the elevation along it increases, whereas in downgrade curve, the elevation decreases.

2.7.1 Types of vertical curves

Vertical curves are usually parabolic, primarily because its shape provides a transition and also computationally efficient. The characteristic of a parabolic curve is that the gradient changes from point to point but the rate of change in grade remains constant. Hence, for finding the length of the vertical curve, the rate of change of grade should be an essential consideration as this factor remains constant throughout the length of the vertical curve. Depending upon the shape of ground profile, a vertical curve may be divided into:

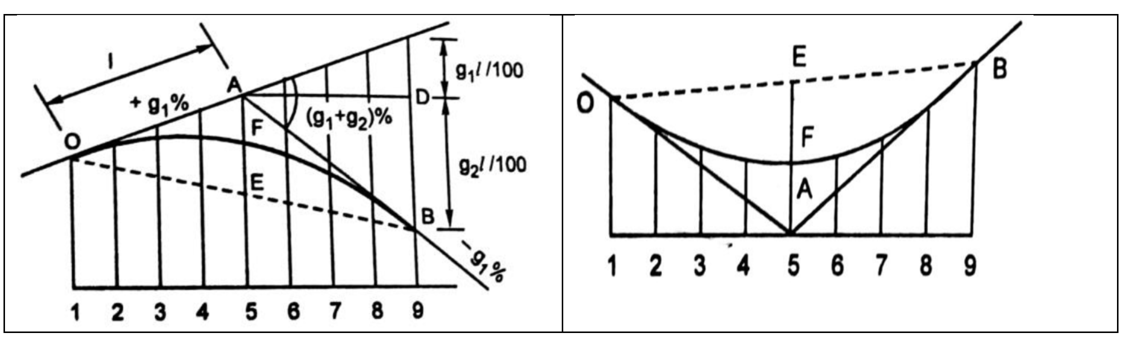
(a) Summit curve: When two grades meet at the summit and the curve will have convexity upwards, the curve is simply referred as summit curve.
(b) Valley (Sag) curve: When two grades meet at the valley (sag) and the curve will have convexity downwards, the curve is simply referred as the valley (sag) curve.

Summit curves are usually provided: (i) when an upgrade is followed by a downgrade, (ii) when a steeper upgrade is followed by a milder upgrade, and (iii) when a milder downgrade is followed by a steeper upgrade. Generally, the recommended rate of change of grade is 0.1% per 30 m at summits and 0.05% per 30 m at sags. During the design of a vertical summit curve, the comfort, appearance, and security of the driver is considered. In addition, the sight distances (SD) must be considered during the design. During movement of a vehicle in a summit curve, there is less discomfort to the passengers because the centrifugal force developed by the movement of vehicle on a summit curve act upwards which is opposite to the direction in which its weight acts. This relieves the load on the springs of the vehicle, so stress developed is less. Therefore, the important part in summit curve design is the computation of length of the summit curve which is done by considering the sight distance parameters.

A valley (sag) curve is usually provided; (i) when a downgrade is followed by an upgrade, (ii) when a steeper downgrade is followed by a milder upgrade, and (iii) when a milder upgrade is followed by a steeper upgrade. Valley curves are those curves which have convexity downwards which requires more considerations. During the daytime, visibility in valley curves is not that much hindered as during the night time; and the only source of visibility in the night becomes headlight of the vehicle and the street lights. In valley curves, the centrifugal force generated by the vehicle moving along a valley curve acts downwards along with the weight of the vehicle and this adds to the stress induced in the spring of vehicle which can cause jerking of the vehicle and discomfort to the passengers. Thus, the most important things to consider during valley curve design are: impact and jerk-free movement of vehicles at the design speed, and the availability of stopping sight distance (SSD) under headlight of vehicles during night.

The summit or valley curves, according to geometrical configuration, can be divided into four categories (Figure 2.20):

(a) An upgrade (+ive gradient) meets by a downgrade (-ive gradient). Such curve is called a convex curve or a summit curve. The change in gradient would be the algebraic difference of the gradients, i.e., [g1 – (-g2)%] or (g1 + g2).
(b) A downgrade (-ive gradient) meets by an upgrade (+ive gradient). Such curve is also called a sag curve or a concave curve. The change in gradient would be the algebraic difference of the gradients, i.e., [(g2 – (-g1)%] = (g1 + g2).
(c) An upgrade (+ive gradient) meets by another upgrade (+ive gradient). Such curve is also called a sag curve or a concave curve. The change in gradient would be the algebraic difference between the gradients [i.e., (g2 – g1)%].
(d) A downgrade (-ive gradient) meets by another downgrade (-ive gradient). Such curve is also called a summit curve or a convex curve. The change in gradient is the algebraic difference between the gradients, i.e., (g2 – g1)%.



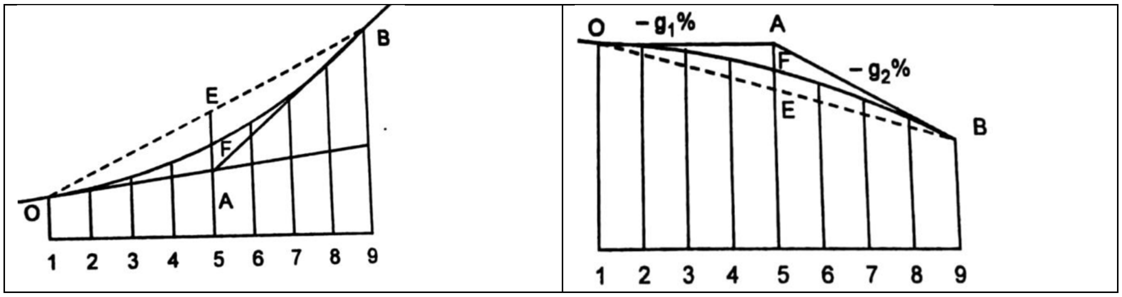


Figure 2.20 Types of vertical curves

The permissible rate of change in gradient for first class railways is recommended as 0.06% per 20 m station at summits and 0.05% per 20 m station for sags. For second class railways, permissible rate of change of gradient is 0.12% per 20 m station at summits and 0.1% per 20 m station for sags. For small gradients, there is no noticeable difference between a parabola and a circular arc. Suppose the gradient at the beginning of a summit curve is 1.20% and if the rate of change of gradient is 0.05% per 20 m station, the gradients at the various stations will be as given in Table 2.1.

Table 2.1 Distance as per change in 0,05% gradient

Station	**Distance from the beginning of the vertical curve (m)**	**Gradients (%)**
1	0	1.20
2	20	1.15
3	40	1.10
4	60	1.05
5	80	1.00

2.7.2 Elements of a vertical parabolic curve

Vertical curves are used to provide gradual change between two adjacent vertical grade lines. The curve used to connect the two adjacent grades is parabola. Parabola offers smooth transition because its second derivative is constant. For a downward parabola with vertex at the origin, the standard equation is-

$x^2 = -4ay$

or $y = -x^2 / 4a$ (2.62)

The first derivative is the slope of the curve.

$y' = -x/ 2a$

The value of y' above is linear, thus the grade (slope) for a summit curve is downward and linear. The second derivative is obviously constant.

$y'' = -1/ 2a$ (2.63)

which is interpreted as rate of change of slope. This characteristic made the parabola the preferred curve because it offers constant rate of change of slope.

The *symmetrical parabolic curve* does not necessarily mean the curve is symmetrical at *L*/2, it simply means that the curve is made up of single vertical parabolic curve. Using two or more parabolic curves placed adjacent to each other is called *unsymmetrical parabolic curve*. Figure 2.21 is a vertical summit curve. The same elements hold true for a vertical sag (valley) curve.

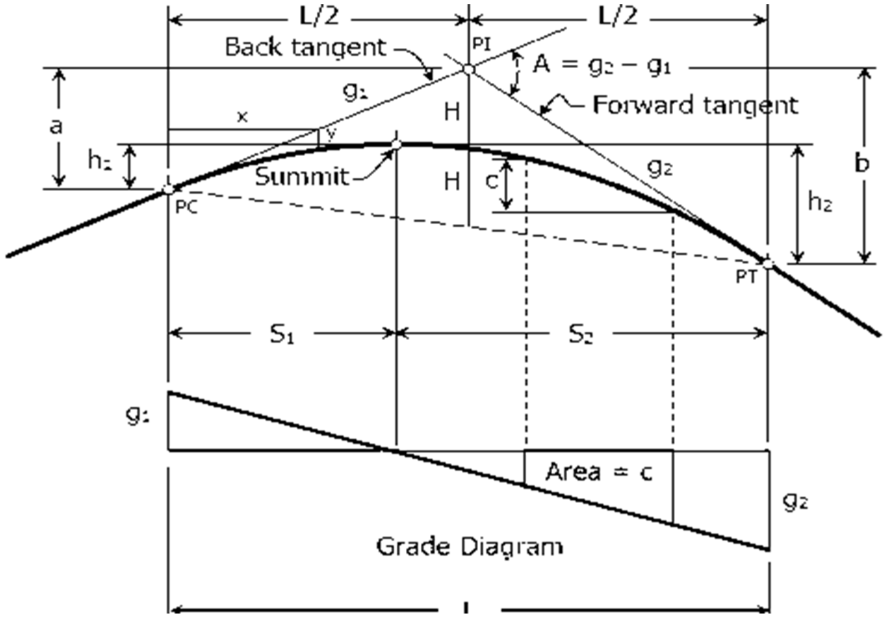


Figure 2.21 Elements of a summit vertical curve

If PC is the point of curvature, also called as BVC (beginning of vertical curve), PT is the point of tangency, also called as EVC (end of vertical curve), PI is the point of intersection of the tangents, also called PVI (point of vertical intersection), L is the length of parabolic curve, it is the projection of the curve onto a horizontal surface which corresponds to the plan distance, S_1 is the horizontal distance from PC to the highest (lowest in case of sag) point of the summit (or sag) curve, S_2 is the horizontal distance from PT to the highest (lowest in case of sag) point of the summit (or sag) curve, h_1 is the vertical distance between PC and the highest (lowest in case of sag) point of the summit (or sag) curve, h_2 is the vertical distance between PT and the highest (lowest in case of sag) point of the summit (or sag) curve, g_1 is the grade (in percent) of back tangent (tangent through PC), g_2 is the grade (in percent) of forward tangent (tangent through PT), A is the change in grade from PC to PT, a is the vertical distance between PC and PI, b is the vertical distance between PT and PI, H is the vertical distance between PI and the curve, then-

The length of parabolic curve L is the horizontal distance between PI and PT. PI is midway between PC and PT. The curve is midway between PI and the midpoint of the chord from PC to PT. The vertical distance between any two points on the curve is equal to area under the Figure. The vertical distance c is area. The grade of the curve at a specific point is equal to the offset distance in the grade diagram under that point.

Rise = run × slope

$a = g_1 L / 2$ (2.64)

$b = g_2 L / 2$ (2.65)

Neglecting the sign of g_1 and g_2 -

$S_1 = (g_1 L) / (g_1 + g_2)$ (2.66)

$S_1 = (g_2 L) / (g_1 + g_2)$ (2.67)

Vertical distance = shaded area under the diagram

$h_1 = g_1 S_1 / 2$ (2.68)

$h_2 = g_2 S_2 / 2$ (2.69)

Other formulas could be-

H= L (g1+g2) / 8 (2.70)

$x^2 / y = (L/2)^2 / H$ (2.71)

2.7.3 Characteristics of a vertical curve

The characteristic of a parabolic curve is that the gradient changes from point to point but the rate of change in grade remains constant. Hence, for finding the length of the vertical curve, the rate of change of grade should be an essential consideration as this factor remains constant throughout the length of vertical curve. Generally, the recommended rate of change of grade is 0.1% per 30 m at summits and 0.05% per 30 m at sags.

In Figure 2.22, AB and BC are two gradient lines intersecting at point B. A vertical curve (T_1FT_a) is to be introduced between these two gradients.

If g_1% is the gradient of line AB (In this case, it is +ve), g_2% is the gradient of line BC (In this case, it is -ve), r is the rate of change of grade, T_1 and T_2 are the tangent points at the beginning of the curve and at the end of the curve, T_1T_2 is the line joining the points of tangency or chord of the curve, E is mid-point of the chord of the curve, and F is the mid-point of the curve, then-

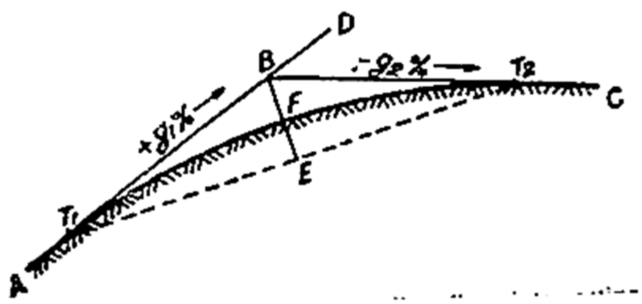


Figure 2.22 Vertical summit curve

(i) It is assumed that length of the vertical curve is equal to the length of the two tangents i.e., $T_1FT_2 = BT_1 + BT_2$

Length of the vertical curve, $L = \dfrac{\text{algabraic difference of two grade}}{\text{rate of change of grade}}$

$$= \frac{g_1 - g_2}{r} \tag{2.72}$$

The length of the curve on either side of the intersection point is also assumed to be equal i.e., half of the length on one side and the remaining half on other side of the intersection point.

(ii) Chainage of beginning of the curve (T_1) = Chainage of B – *L/2* (2.73)

(iii) Chainage of end of the curve (T_2)

= Chainage of the intersection point + half the length of vertical curve

= Chainage of B + *L/2* (2.74)

(iv) RL of T_1 = RL of B $\pm$level difference between T_1 and B

(+ve sign is used, if g_1 is –ve and vice versa). In this case g_1 is +ve, so -ve sign is to be used.

$$RL\ of\ T_1 = RL\ of\ B - \frac{L}{2} \times \frac{g_1}{100} \tag{2.75}$$

(v) RL of T_2 = RL of B $\pm$level difference between B and T_2

(+ve sign is used if g_2 is +ve, and –ve if g_2 is -ve). In this case g_2 is –ve, so -ve sign is to be used.

$$RL\ of\ T_2 = RL\ of\ B - \frac{L}{2} \times \frac{g_2}{100} \qquad (2.76)$$

(vi) RL of E = $\frac{1}{2}(RL\ of\ T_1 + RL\ of\ T_2)$ (2.77)

(vii) It is well known property of parabola that mid-point E of the chord T_1T_2 is situated on the vertical through the point of intersection B of the two tangents, and that mid-point F of the vertical curve is midway between these two points.

BF = FE

RL of F = $\frac{1}{2}$ (RL of E + RL of B) (2.78)

(viii) The curve length of either side of the intersection point is divided into suitable number of parts (15 m, 20 m or 30 m) and the RL of the points on the curve may be calculated by adding algebraically the tangent corrections (tangents offsets) to the RLs of the corresponding points on curve.

The RL of a point at a distance *x* on the curve = RL of the point at a distance *x* along the tangent ±tangent correction (y_x).

(a) The RLs of points along the tangent can be determined from the known values of RL of the tangent point or intersection point and the gradient of tangent lines.

RL of a point at a distance *x* from the tangent point = RL of the tangent point $\pm \frac{g}{100} \times x$

(2.79)

(b) Tangent correction (tangent offset) for a point at a distance x along the tangent.

$$y_x = \frac{g_1 - g_2}{400} \times \frac{x^2}{half\ the\ length\ of\ the\ curve} \qquad (2.80)$$

Unit Summary

This unit discusses two broad type of curves used in roads and railways; horizontal curves and vertical curves. Undwerstanding the importance of curves is neceassry as they are frequently employed due to topography and other features on the ground. Various types of horizontal curves: Simple circular curves, Compound curves, Reverse curves and Transition curves are explained. The characterstics of each curve are given. Mathematical relationships are derived so that the neceassary components of different types of curves are computed and curves established on the ground, mainly using angular and linear measurements. The super-elevation plays an important role while the vehicle is moving a curve, so it is necessary to provide while a curve is laid out. The gradient of vertical curves is quite significant which determines the stopping sight distance at a particular speed of the vehicle. plays an important role.

Solved Examples

Example 2.1:

A circular curve has 300 m radius and 60° deflection angle. Determine its degree by (a) arc definition and (b) chord definition of standard length 30 m. Also compute the (i) length of curve, (ii) tangent length, (iii) length of long chord, (iv) mid-ordinate and (v) apex distance.

Solution:
R = 300 m, D = 60°
(a) Arc definition: s = 30 m,
R = (s /D_a)(180/ π)
300 = 30 *180 / (D_a π)
D_a = 5.730 Ans.

(b) Chord definition: R sin (D_c/ 2) = s /2
300 sin D_c/ 2 = 30 / 2
D_c = 5.732 Ans.

(i) Length of the curve: L = R Δ (π/180)
L = 300 * 60 (Δ/180) = 314.16 m Ans.

(ii) Tangent length: T = R tan Δ/2
T= 300 tan 60/2 = 173.21 m Ans.

(iii) Length of long chord: L_c = 2 R sin Δ /2
L_c = 2 * 300 sin 60/ 2 = 300 m Ans.

(iv) Mid-ordinate: M = R (1- cos Δ/2)
= 300 (1- cos 60/2) = 40.19 m Ans.

(v) Apex distance: E = R (sec Δ/2 – 1)
= 300 (sec 60/2) = 46.41 m Ans.

Example 2.2:
If the approximate perpendicular offset for the mid-point of the circular curve deflecting through 76^0 38' is 96.1 m, calculate the radius of the curve.

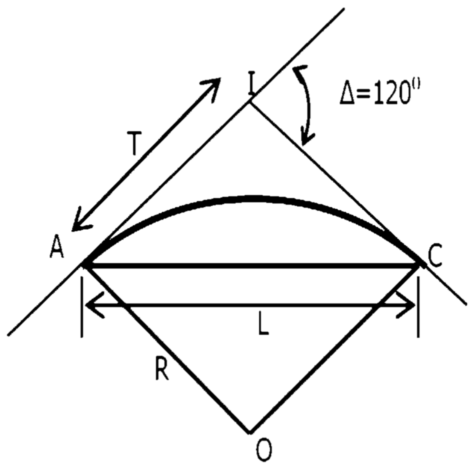
Solution:
Δ = 76^0 38', Ox = 96.1 m
Using Perpendicular offset method:
$O_x = R - \sqrt{[R^2 - (x)^2]}$
The distance ‘x’ from T_1 for locating the apex point = R sin Δ/2
= R sin 76^0 38' / 2 = 0.62R m
Now
$96.1 = R - \sqrt{[R^2 - (0.62R)^2]} = 0.215\ R$
R = 96.1/ 0.215 = 446.98 m

Example 2.3:
Determine the ordinates of the points on a circular curve having a long chord of 100 m and a versed sine of 5 m. The ordinates are to be measured from the long chord at an interval of 10 m.

Solution:
Length of long chord (L) = 100 m, Versed sine (O_0) = 5 m, Interval = 10 m
The versined or mid-ordinate
$O_0 = R - \sqrt{[R^2 - (L/2)^2]}$
$5 = R - \sqrt{[R^2 - (100/2)^2]}$

R = 252.5 m
Ordinates at intervals of 10 m
$O_0 = \sqrt{[R^2 - (x)^2]} - (R - O_0)$

Example 2.4:
Two straight alignments AI and IC of a road intersect at I at a chainage (250 chainage +15 links), angle of deflection being 120^0. Calculate the chainages of the point of commencement and the point of tangency if the radius of the right hand circular curve is 200 m. Assume the length of the chain as 30 m.



Solution
Chainage at the point of intersection = (250+15) i.e., 250 chains and 15 links.
Length of one chain = 30 m so chainage at the point of intersection = 250 x 30 + 15 x 0.2 = 7503 m
Deflection angle (Δ) = 1200, Radius of curve = 200 m.
Tangent Length T= R tan Δ/2
= 200 tan 120/2 = 346.41m
Length of the curve (l)= πRΔ/ 180 = 200 * 120 π/ 180 = 418.88 m
Chainage at point of curve (T_1) = Chainage at the point of intersection – tangent length
= 7503 – 346.41 = 7156.59 m
Chainage at point of tangency (T_2) = Chainage at point of curve (T_1) + Length of curve
= 7156.59 + 418.88 = 7575.47 m

Example 2.5:
Two straights AC and CB intersect at C at a chainage of 86.22 chains at a deflection angle of 62^0. They are to be smoothly connected by a simple curve of radius 12 chains. Find the tangent length, length of curve and the chainages of the starting and end points of the curve. Also, find the length of the long chord.

Solution:
Chainage at the point of intersection (C) = 86.22 chains, Deflection angle (Δ) = 62^0, Radius of curve = 12 chains.

Tangent Length T= R tan Δ/2 = 12 tan 62/2 = 7.21 chains.
Length of the curve (*l*) = πRΔ/ 180 = 12 * 62 π/ 180 = 12.985 chains
Chainage at point of curve (T_1) = Chainage at the point of intersection – tangent length
= 86.22 – 7.21 = 79.01 chains

Chainage at point of tangency (T_2) = Chainage at point of curve (T1) + Length of curve
= 79.01+12.985 = 91.995 chains
Length of Long chord (L) = T_1T_2 = 2R sin Δ/2 = 2 * 12 chains sin 62/2 = 12.361 chains

Example 2.6:
Two straights intersect at a chainage 2056.44 m with an angle of intersection is 130^0. If the radius of the simple curve is 50 m, set out the curve by offsets from long chord at 5 m interval, and find the following:
(i) Chainage of the point of commencement
(ii) Chainage at point of tangency
(iii) Length of the long chord

Solution:
Chainage at the point of intersection = 2056.44 m, Angle of intersection = 130^0 , Radius of curve (R) = 50 m
Deflection angle (Δ) = 180^0 - 130^0 = 50^0
Tangent Length T= R tan Δ/2 = 50 tan 50/2 = 23.32 m
Length of the curve (*l*) = πRΔ/ 180 = 50 * 50 π/ 180 = 43.63 m

Chainage at point of curve (T_1) = Chainage at the point of intersection – tangent length
= 2056.44 – 23.32 = 2033.12 m
Chainage at point of tangency (T2) = Chainage at point of curve (T_1) + Length of curve
= 2033.12 + 43.63 = 2076.75 m
Length of Long chord (L) = T_1T_2 = 2R sin Δ/2 = 2 * 50 sin 50/2 = 42.26 m

Starting from centre of long chord, offset at 5 m interval are calculated for half of the long chord, i.e., 42.26/2 = 21.13m
$y = \sqrt{(R^2 - x^2)} - \sqrt{[R^2 - (L/2)^2]}$
$y = \sqrt{(50^2 - x^2)} - \sqrt{[50^2 - (42.26/2)^2]}$
$y = \sqrt{(50^2 - x^2)} - 45.32$
$y_0 = \sqrt{(50^2 - 0^2)} - 45.32 = 4.68$ m
$y_5 = \sqrt{(50^2 - 5^2)} - 45.32 = 4.43$ m
$y_{10} = \sqrt{(50^2 - 10^2)} - 45.32 = 3.67$ m
$y_{15} = \sqrt{(50^2 - 15^2)} - 45.32 = 3.38$ m
$y_{20} = \sqrt{(50^2 - 20^2)} - 45.32 = 0.51$ m
$y_{21.13} = \sqrt{(50^2 - 21.13^2)} - 45.32 = 0$ m

The other half of the curve is symmetrical so there is no need to calculate for the other half.

Example 2.7:
Two roads having a deflection angle of 45° at apex point V whose chainage of apex point is 1839.2 m, are to be joined by a 200 m radius circular curve. Compute the necessary data to set out the curve by: (a) ordinates from long chord at 10 m interval (b) method of bisection to get every 8^{th} point on curve (c) radial and perpendicular offsets from every full 30 m length along tangent. and (d) offsets from chord produced.

Solution:
R = 200 m and D = 45°
Length of tangent = 200 tan 45/2 = 82.84 m
Chainage of T_1 = 1839.2 – 82.84 = 1756.36 m

Length of curve = R * 45 (π/180) = 157.08 m
Chainage of forward tangent T_2 = 1756.36 + 157.08 = 1913.44 m

(a) By offsets from long chord:
Distance of DT = L/2 = R sin Δ/ 2
= 200 sin 45/2 = 76.54 m
Measuring 'x' from D,
$y = \sqrt{(R^2 - x^2)} - \sqrt{[R^2 - (L/2)^2]}$
At x = 0,
$O_0 = 200 - \sqrt{(200^2 - 76.54^2)}$
= 15.22 m
$O_1 = \sqrt{(200^2 - 10^2)}$ = 14.97 m
$O_2 = \sqrt{(200^2 - 20^2)}$ = 14.22 m

$O_3 = \sqrt{(200^2 - 30^2)}$ = 12.96 m
$O_4 = \sqrt{(200^2 - 40^2)}$ = 11.18 m
$O_5 = \sqrt{(200^2 - 50^2)}$ = 8.87 m
$O_6 = \sqrt{(200^2 - 60^2)}$ = 6.01 m
$O_7 = \sqrt{(200^2 - 70^2)}$ = 2.57 m

At T_1, O = 0.00

(b) Method of bisection:
Central ordinate at D = R (1 - cos Δ/2)
= 200 (1 – cos 45/2) = 15.22
Ordinate at D_1 = R (1 - cos Δ/2)
= 3.84 m
Ordinate at D_2 = R (1 - cos Δ/2)
= 0.96 m

(c) Offsets from tangents:
Radial offsets: $O_x = \sqrt{(R^2 + x^2)} - R$
Chainage of T_1 = 1756.36 m
For 30 m chain, it is at = 58 chains + 16.36 m
x_1 = 30 – 16.36 = 13.64
x_2 = 43.64 m
x_3 = 73.64 m and
the last is at x4 = tangent length = 82.84 m
$O_1 = \sqrt{(200^2 + 13.64^2)} - 200$ = 0.46 m
$O_2 = \sqrt{(200^2 + 43.64^2)} - 200$ = 4.71 m
$O_3 = \sqrt{(200^2 + 73.64^2)} - 200$ = 13.13 m
$O_4 = \sqrt{(200^2 + 82.84^2)} - 200$ = 16.48 m

(d) Offsets from chord produced:
Length of first sub-chord = 13.64 m = C_1
Length of normal chord = 30 m = C_2
Since length of chain is 157.08 m, so $C_2 = C_3 = C_4 = C_5$ = 30 m
Chainage of forward tangent = 1913.44 m = 63 chains + 23.44 m
Length of last chord = 23.44 m = C_n = C6

$O_1 = C^2_1 / 2 R = 13.64^2 /2 * 200 = 0.47$ m
$O_2 = C_2 (C_1 + C_2)/ 2R = 30^2 (13.64 + 30) / 400 = 3.27$ m
$O_3 = C^2_2/ 2R = 30^2 / 400 = 4.5$ m $= O_4 = O_5$
$O_6 = C_n (C_{n-1} + C_n)/ 2R = 23.44 (30 + 23.44) / 400 = 3.13$ m

Example 2.8:
Two roads meet at an angle of 127^0 30'. Calculate the necessary data for setting out a curve of 15 chains radius to connect the two straight portions of the road if the curve is to be set out the curve by chain and offsets only. Assume the length of chain as 20 m.

Solution:
Angle of intersection = 127^0 30', Deflection angle (Δ) = $180^0 - 127^0$ 30' = 52^0 30', Length of chain = 20 m, Radius of curve = 15 chains x 20 = 300 m.

Tangent Length T= R tan $\Delta/2$ = 300 tan 52^0 30' / 2 = 147.94 m

(a) Radial offset method:
$O_x = \sqrt{(R^2 + x^2)} - R$
First half of the curve is set from point of curve (T_1)
Assuming interval as 20 m
$O_{20} = \sqrt{(300^2 + 20^2)} - 300 = 0.67$ m
$O_{40} = \sqrt{(300^2 + 40^2)} - 300 = 2.66$ m
$O_{60} = \sqrt{(300^2 + 60^2)} - 300 = 5.94$ m
$O_{80} = \sqrt{(300^2 + 80^2)} - 300 = 10.48$ m
$O_{100} = \sqrt{(300^2 + 100^2)} - 300 = 16.23$ m
$O_{120} = \sqrt{(300^2 + 120^2)} - 300 = 23.11$ m
$O_{140} = \sqrt{(300^2 + 140^2)} - 300 = 31.06$ m
$O_{147.94} = \sqrt{(300^2 + 147.94^2)} - 300 = 34.49$ m
The second half of the curve may be set from point of tangency (T_2)

(b) Perpendicular offset method:
$O_x = R - \sqrt{[(R^2 - x^2)]}$
First half of the curve is set from point of curve (T_1)

$O_{20} = R - \sqrt{[(R^2 - 20^2)]} = 0.67$ m
$O_{40} = R - \sqrt{[(R^2 - 40^2)]} = 2.68$ m
$O_{60} = R - \sqrt{[(R^2 - 60^2)]} = 6.06$ m
$O_{80} = R - \sqrt{[(R^2 - 80^2)]} = 10.86$ m
$O_{100} = R - \sqrt{[(R^2 - 100^2)]} = 17.16$ m
$O_{120} = R - \sqrt{[(R^2 - 120^2)]} = 25.05$ m
$O_{140} = R - \sqrt{[(R^2 - 140^2)]} = 34.67$ m
$O_{147.94} = R - \sqrt{[(R^2 - 147.94^2)]} = 39.01$ m
The distance 'x' from T_1 for locating the apex point = R sin $\Delta/2$
x = 300 sin 52^0 30' = 132.69 m
$O_{132} = 300 - \sqrt{[(300^2 - 132.69^2)]} = 30.94$ m
Second half of the curve may be set from point of tangency (T_2)
Example 2.9:
Compute the necessary data for setting out a circular curve of 300 m radius by deflection angle method. The peg interval is 30 m Two tangents intersect at the chainage 1190 m, with the 36° deflection angle.