Hydraulic Engineering
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Lecture-23
Dimensional Analysis and Hydraulic Similitude

Welcome. This week we are going to study a topic called dimensional analysis. So, this is week number 5, of this course, that is, hydraulic engineering, we straight away, get going now.

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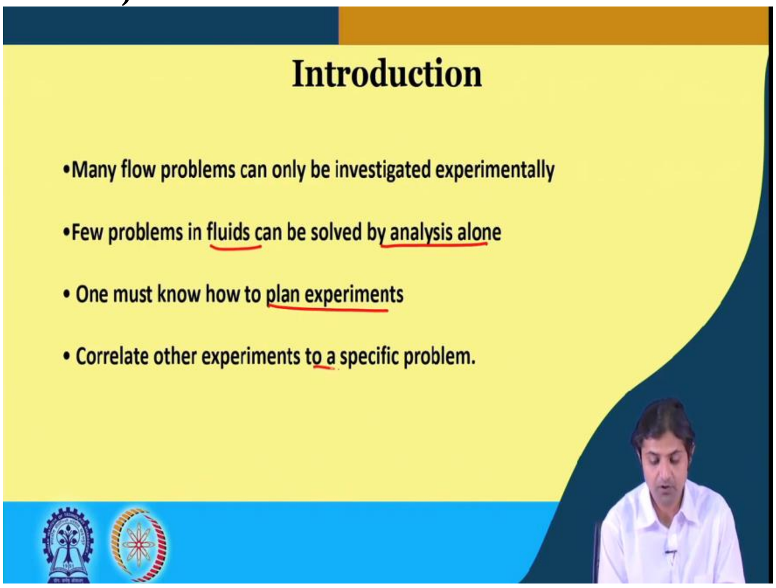

So, as always, we have to introduce the topic to you. And one of the important points that as a student of this course, you µst know that, that most of the problems, actually many of the problems in fluid mechanics can only be investigated experimentally. And there are only some problems in fluid that can be solved by analysis alone. Analysis as in numerical analysis or analytical solutions and most of them rely on the experiments.

Therefore, it is required that a student of hydraulic engineering in their research they µst know how to plan the experiments. And after the experiment is conducted, an important step again is, that we have to correlate other experiments to a specific problem.

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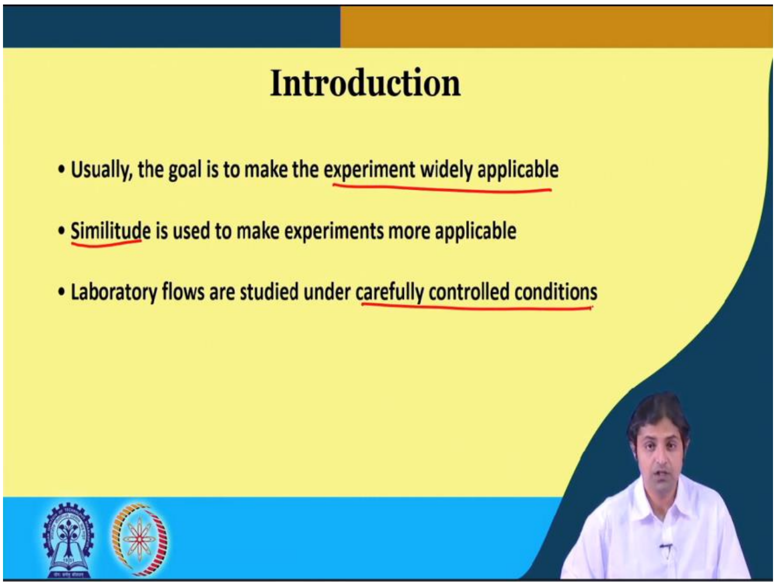

So, what is the most important goal of this exercise? The goal is to make those experiments that we have done, widely applicable, not only for the conditions that the experiment was done, but so that those experiments can be applied to the other scenarios, as well. And one process to do that is called similitude. So, similitude is a process which is used to make experiments more applicable.

So, in this week, this module, where we are going to investigate the dimensional analysis, we are going to see in detail what similitude is, but for now, just take that it is a process which is used to make experiments more applicable. We know that the laboratory flows, I mean, the experiments that are in lab are studied under carefully controlled conditions. In nature, it might not be the case. In nature, there is no control over the conditions, but in laboratory we can actually control the conditions.

The conditions like air temperature or even the velocity can be controlled. Whatever we want, I mean, even the sediment sizes, the grain, in real nature, if you go to a riverbed the sediment will be comprising of a different grain sizes. Here, an experiment if we want we can have a uniform set of grain sizes, for example.

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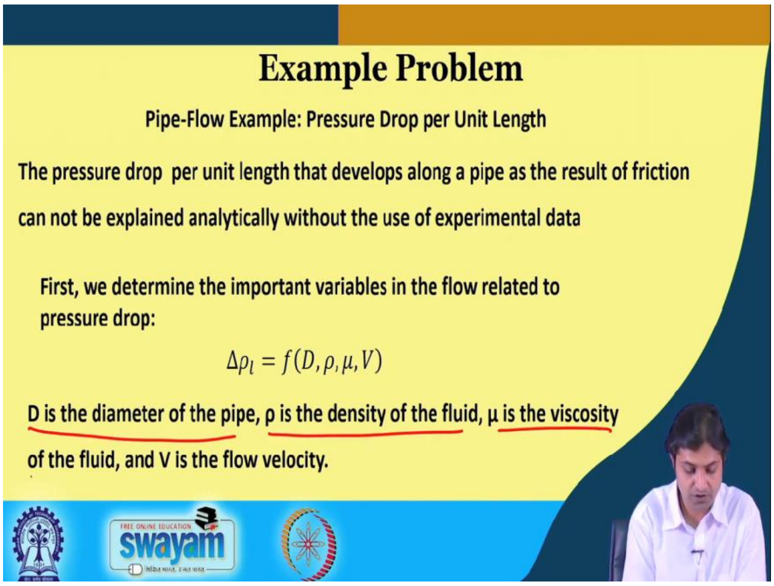

So, let us take an example problem. It is a very, very famous problem, which is an example of the pipe flow. And what we have to determine? We have to determine, pressure drop per unit length. So, we will explore, in principle, how these things can be done using experiments, for example. So, the pressure drop per unit length of the pipe that develops along it, is a result of friction. And this phenomenon cannot be explained analytically without the use of experimental data.

So, many people have tried to explain it on the basis of analytical results and reticulous more called mathematical analysis. But without the use of experimental data, this phenomenon cannot be actually explained in full detail. So, what do we do? First, we determine the important variables of the flow related to the pressure drop. So, we µst know, what are the flow parameters, for example? Flow parameters or important variables here, what we call. That is related to this pressure drop.

So, it has, I mean, when it comes to our mind, we can write that pressure P per unit length. So, this is length, can be a function of, this is D, that is, it is a function of diameter of the pipe, ρ is the density of the fluid, viscosity of the fluid and of course, it should also depend upon the velocity of the flow. This is the most simplified assumption.

So, the pressure per, pressure drop per unit length should depend upon the diameter of the pipe, it should also depend upon the density of the liquid or the fluid, it should also depend upon the viscosity of the liquid or fluid and also the velocity of the flow. These are some things that comes to our mind. As I told you, D the diameter of the pipe, ρ is the density of the fluid, μ the viscosity of the fluid and V is the flow velocity.

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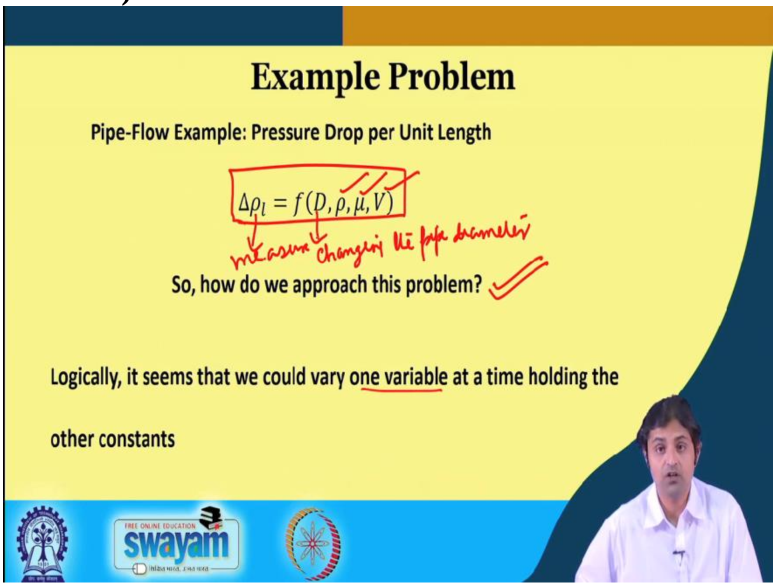

So, the example problem we are continuing with the discussion. So, as we wrote, I am writing this expression again, that pressure drop per unit length is function of these 4 variables. So the question is, how do we approach this problem? That is the key here. So, normally if we want to conduct experiments, it is, I mean, it is very logical that we can vary one variable at a time and hold the other constant.

So, we have an apparatus equipment setup in the lab. So, for a particular value of density, what a particular value of μ and for a particular flow velocity we can keep on changing the pipe diameter. This is one and we keep everything else in constant and measure this. The other, some other experiments could be we can start varying. So, density of the liquid, we can use a different liquid altogether.

So, that will change its density and kinematic viscosity as well. And the other set could be, we can actually change the flow velocities and keep on conducting many experiments.

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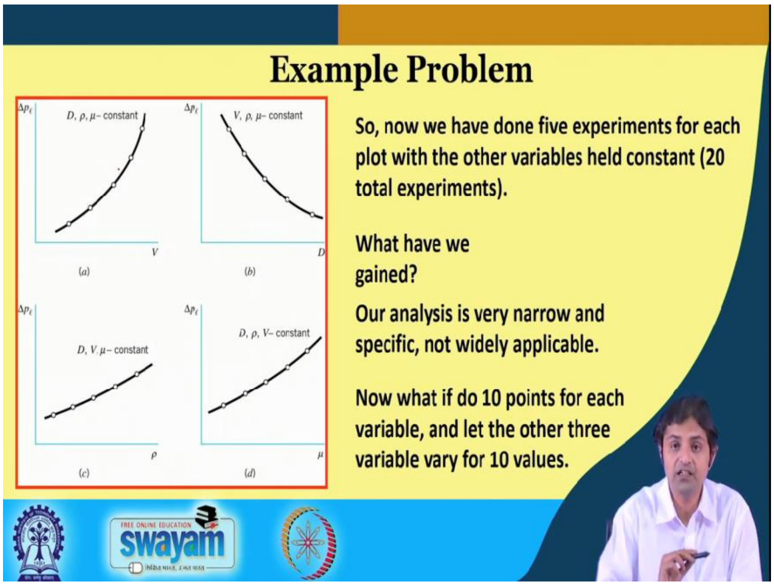

So, this, the hand side is the plot. I am going to explain you what those plot are. So, if you look here, the graph a, what it shows is, that we have varied the flow velocity, we have kept diameter ρ and μ as constant and start measuring the pressure drop per unit length using our apparatus. What we find out is, the pressure drop per unit length is increasing as the velocity is increase. This is a sort of a curve that we are getting.

In the second b, what is happening is, we are holding velocity ρ μ constant and we are varying the pipe diameter, as we discussed in the last slide and start measuring. So, we see, both in a and b, we have done 1 experiment, 2 experiment, 3 experiments 4, 5. In c, here also 1, 2, 3, 4, 5, here also in the graph c, if we see we have varied only the density and constant D, V and μ. I mean, you would have a very valid question how can, we keep μ, for example, constant and vary ρ.

There are because in laboratory we can control the conditions and therefore, it might be possible. So, just taking a hypothetical scenario by for example, if we vary ρ and similarly, we have got here 5 points 3, 4, 5 and in the last we keep diameter ρ and V constant and vary the viscosity of the fluid. And we see again, we have got 1, 2, 3, 4 and 5.

So, what we see, we have now done 5 experiments, for each plot and the while the other things were constant. Therefore, the total number of experiments have been done as 20. So, what have we gained from these experiments? I will just take away this. What have we gained? We know that our analysis is very narrow and specific and it is not widely applicable because we have only 20 experiments and for a particular velocity, I mean, for a particular pipe diameter ρ and μ. In the other, we have a particular V, ρ and μ so and so on. Now, if we decide, we will do 10 points for each variable and let the other 3 variables vary for 10 values.

So, we do so, we do 10 points for each variable. So, instead of 5 here, we do 10 and the other 3 variable can also vary for 10 different values. So, this will

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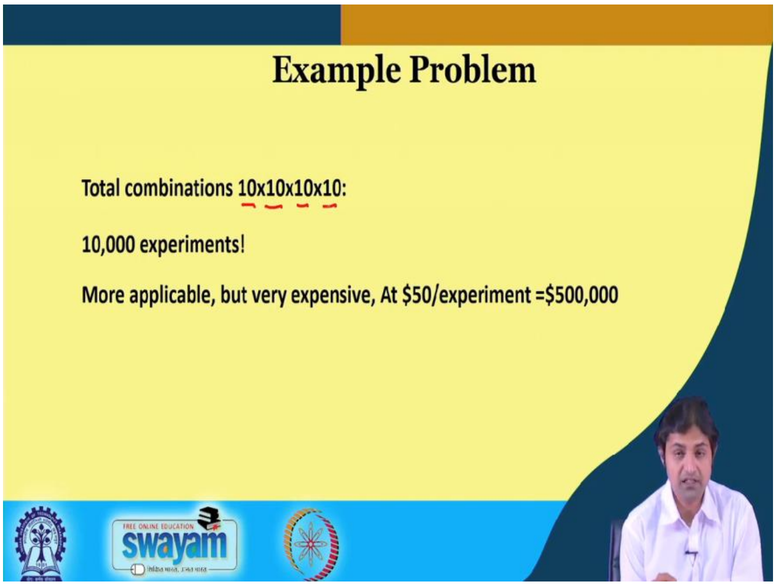

So, total combination because there are 4 variables so, what is going to be our number of experiments? 10 into 10 into 10 into 10. So, there will be 10,000 experiments that we have to conduct and, you know, conducting 10,000 experiments to make it more widely applicable is not a very good idea. I mean, this is definitely possible, but normally it is not preferred. And one of the other, you know, experiments are expensive, very expensive.

So, each of the experiment will have a cost implication. In US, one experiment typically, let us say in this case is cost 50 dollar per experiment. So, for 10,000 experiment, we need at least half a million US dollars. But we have done more experiments and therefore, our, I mean, our results that we have got would be more widely applicable because there are more number of sets, data sets.

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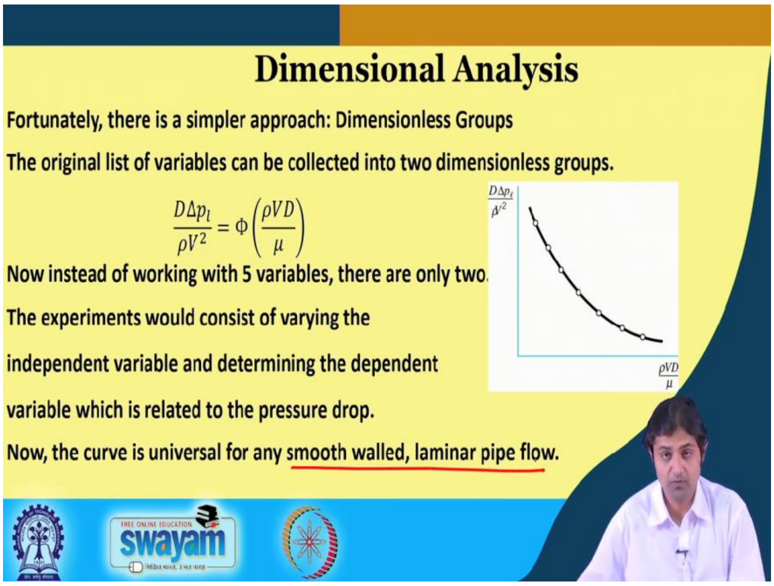

Fortunately, to avoid this, to have, to do, to be able to do less number of experiments, there is a µch, µch simpler approach called dimensionless groups or the dimensional analysis. So, this makes our life easy and is therefore the topic for this week's module. So, the original list of variables can, I mean, what happens in dimensional analysis or you know dimensionless groups. The original list of variables can be collected in 2 dimensionless groups.

I mean, it could be more but let us set two dimensionless group. So, for pipe it has been found out that this is one dimensionless group and this is the other dimensionless group. We will actually go and see how this comes using dimensional analysis but for now for to explain you, I mean, to give you a good introduction, I have just written it down. This comes out that this group, dimensionless group is a function of the other dimensionless group.

Now, earlier, how many variables we had? We had, delta p l was a function of D, ρ, µ, V. How many? 1, 2, 3, 4, 5, so, earlier there were 5 variables. But in this case, if we assume this as one group, and this is one group, how many variables? We have only 2 variables. So, having only 2 variables instead of 5, is it better or not? It is definitely µch, µch better than using more number of variables.

And if we plot the curve between $\frac{\rho VD}{\mu}$ and $\frac{D\Delta p_1}{\rho V^2}$, a graph something like this will come and there are only 2 variables, this and this. If we call this a and this is b, so, it is 2 variables a and b. Now, if we try to do experiments, the experiments would consist of varying the independent variable. So, we can vary this one, this is an independent variable and we can determine the dependent variable here which is related to the pressure drop.

So, if we determine this, so, using results obtained in this experiment we can determine the dependent variable and our pressure drop is related to this dependent variable. Another advantage is now; this curve is universal for any smooth walled, laminar pipe flow because we have assumed the assumptions where we did not assume any roughness, so it was a smooth walled, laminar pipe flow. Mean we will come to this there is a couple of modules on pipe flow and open channel flow. Soon we will get some introduction in the next week's starting.

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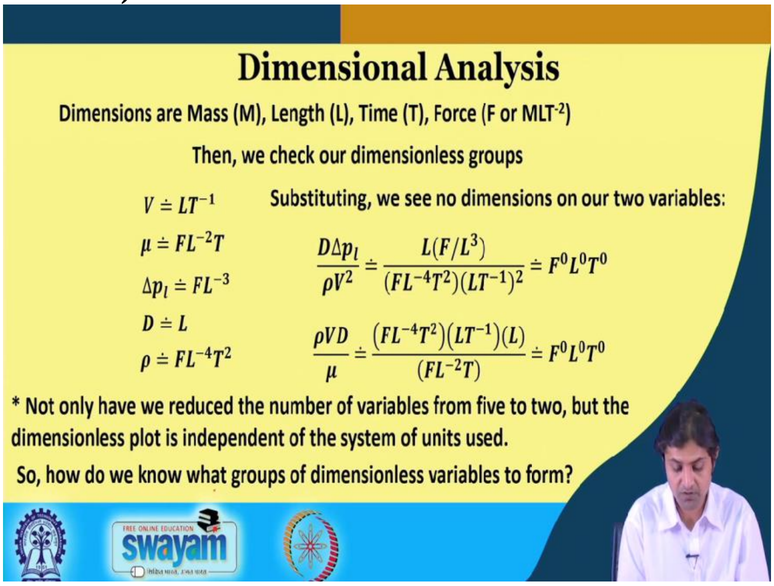

So, getting back to the basics, which you have already learned in your class 10th, I mean 11th and 12th. The basic dimensions are the mass M, length L and time T. And now we can also say force F can also be considered one of the dimensions. However, force is not independent, it is it has MLT - 2. So, you can choose, 2 of these plus force, for example, or 2 of these plus force, or 2 of these plus force. So, choose any 3, these are the 3 basic dimensions. Actually, 4 here, in this case.

Now, going back to the previous problem, we should check our dimensionless group. Were those dimensionless group that we had the 2, were really dimensionless or not? So, the velocity is meters per second or LT^{-1}, it is distance per unit time. So, distance is LT. So, LT^{-1} is the velocity, dimension of velocity. Kinematic, we, in this problem we are writing it in terms of F L and T. So, μ can be written as, the dimensions can be written as $FL^{-2}T$.

Pressure per unit length is FL^{-3} is the pressure and per unit length, so, it becomes FL^{-3} this is very simple to compute. So, you can actually calculate it. D is the dimension of, diameter is the dimension of length, so, it is L. And our last is the density, in terms of force, it can be written as $FL^{-4}T^{2}$, you actually you should verify this at home. After this lecture is finished after going through this lecture or even, you can go and try to find it, in terms of F.

So, we substitute, into our dimensionless group we will see, so one of first, this was dimensionless group 1. So, D is length. I will just take the eraser and instead use laser pointer. So, D is length, delta pl is F / L cube, from here. P is FL – 4T square and this is LT - 1 whole, LT - 1 to the whole square. And, so, if you start doing, you will get, F to raise to the power 0, L to the power 0, t to the power 0. So, I will just. So, we had no dimensions here.

Similarly, we do for ρ VD / μ, ρ's dimension is FL - 4T square, velocity dimension is LT - 1 and diameter is L, where μ is FL - 2T. For, we have, I mean, our dimensions we have already written. Actually the first step in any dimensionless question is that you write the dimensions of the most basic variables and that is what we have done here. And if you try to, you know, cancel out, we will again get F to the power 0, L to the power 0, T to the power 0. So, basically this means dimensionless.

So, now, we have not only reduce the number of variables from 5 to 2. But the dimensionless plot which we have got in the last slide is independent of the system of units used. Does not matter if we have used the SI system or CGS system or any system, for that matter, because there are no dimensions associated with it. So, the 2 advantages were less variables and very important, that is, independent of unit, these dimensionless groups independent of units.

So, the most important question is, how do we know what groups of dimensionless variable to form? This is quite an important question. I mean, we can form many, you know, groups, but what groups of dimensionless variable to form.

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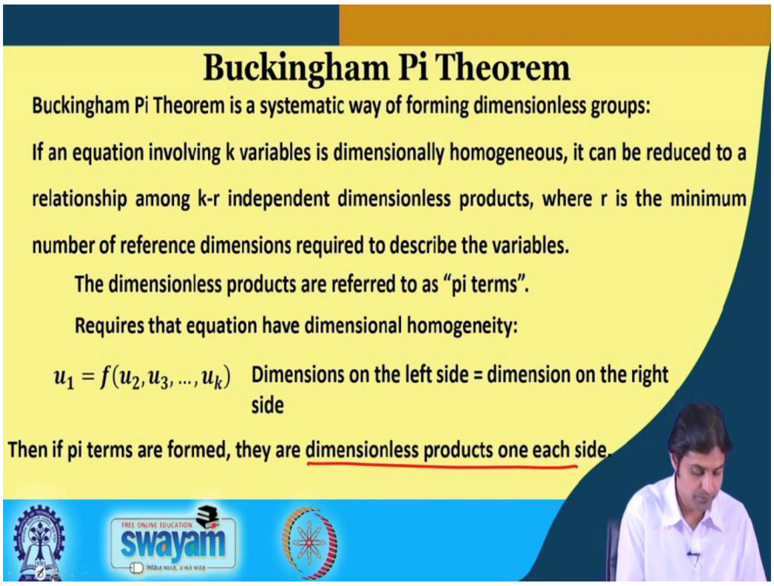

And to answer that we have a theorem called Buckingham Pi theorem which forms the core of this module. So, please understand this theorem very carefully. So, Buckingham Pi theorem is actually a very systematic way of forming dimensionless groups, very systematic way. So, if you follow those steps carefully, you will always be able to solve the problems, any problems related to dimensional analysis.

So, what the theorem says? So, this is the theorem, it says that the, if an equation which involves k variables is dimensionally homogeneous, then it can be reduced to a relationship among k - r independent dimensionless products, where r is the miniµm number of reference dimensions required to describe the variables, . So, again I am taking away the ink. I will write, you know, important I will headline, I mean, I will underline the important things. If there is an equation, an equation that we have been writing before that involves in total k variables.

Of course in equation should be dimensionally homogeneous that is why this is there. So, if there is an equation that consists of k variables, then it that equation can be reduced to k - r independent dimensionless products. Now, what is this r? r is actually the miniµm number of reference dimensions required to describe the variables. I just telling you an example, this r will be either 1, 2 or 3 because there are only 3 dimensions.

I mean, in principle, M,L and T or F,L and T. So, if I mean, we will see question by question and there will be cases in which this r is only 2 because we do not deal with, for example, mass at times. So, k – r, that is, the key, we will go into more detail soon. And these dimensionless products that are referred, these dimensionless products are called as Pi terms.

So, that is called Pi terms and this requires that equations that we are going to solve have dimensional homogeneity.

So, dimensional homogeneity is that all the groups should have same dimensions, you know, the equations on the left hand side and the right hand side. Suppose, there is u 1 is a function of u 2, u 3 up till u k and this means that the dimensions on the left hand side, so, u 1 should be equal to the dimensions on right side and this is called dimensional homogeneity. So, then if Pi terms are formed, they are dimensionless products one each side. So, then if Pi items are formed here, then these will be, you know, dimensionless groups.

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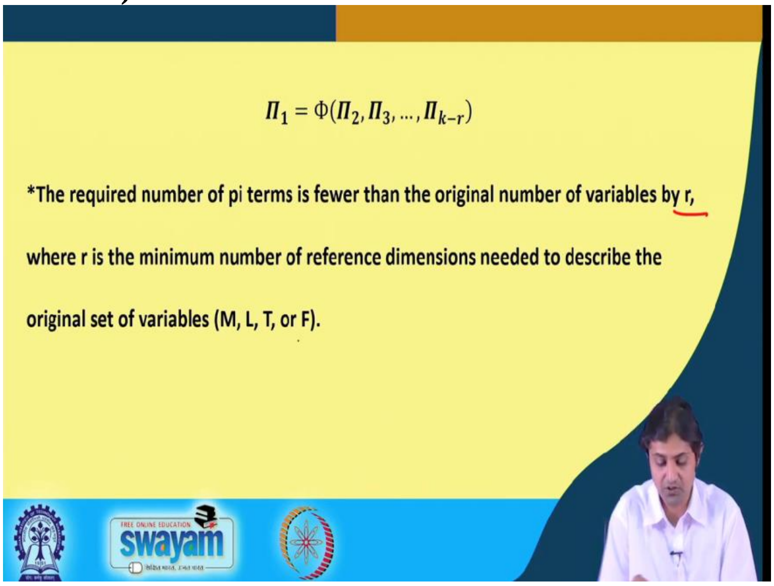

So, Pi 1, this is the first Pi term, so, the final equation is going to be Pi 1 is a function of Pi 2, Pi 3, Pi because this is also dimensionless. Same here, all these things are dimensionless. So, dimensional homogeneity is going to be maintained. So, the required number of Pi terms, it is fewer or lesser than the original number of variables by r. So, k - r, so, the number of dimensionless group would be equal to k - r.

And here, r is the miniμm number of reference dimensions needed to describe the original set of variables. So, M,L,T or F, it could be all, I mean, 3 of them, or 2 of them or could be one of them. You will get to understand this more clearly when we start solving the problems.

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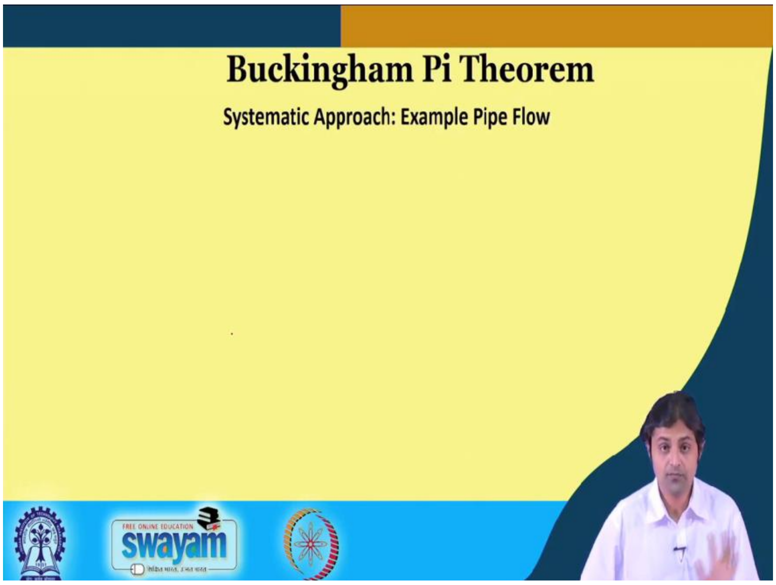

So, there is a systematic approach, which I think I am going to start, you know. I am going to close this lecture here, so that we can start afresh from while we will do this example of pipe flow and we are going to solve it very systematically and one by one, listing all these steps. So, this is for now, I will see you in the next lecture.