

Chapter 7: Two-Port Networks - Basic Concepts and Parameters

7.1 Introduction to Two-Port Networks

- **Definition**:

- A **two-port network** is an electrical circuit with **two pairs of terminals** (input and output ports), used to model amplifiers, filters, and transmission lines.

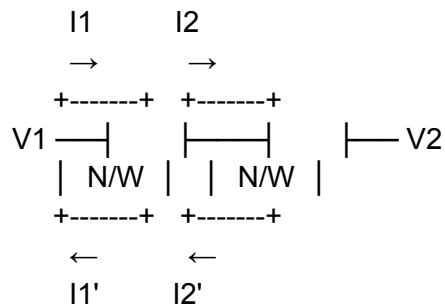
- **Key Assumptions**:

- Linear time-invariant (LTI) system
- No independent sources inside the network
- Port currents satisfy $(I_1 = I_1')$ and $(I_2 = I_2')$ (current entering = current leaving)

7.2 Two-Port Network Representations

7.2.1 General Structure

...



...

7.2.2 Parameter Types

Two-port networks can be described using **six parameter sets**:

1. **Impedance (Z) Parameters**
2. **Admittance (Y) Parameters**
3. **Hybrid (h) Parameters**
4. **Inverse Hybrid (g) Parameters**
5. **Transmission (ABCD) Parameters**
6. **Scattering (S) Parameters (for RF circuits)**

7.3 Impedance (Z) Parameters

7.3.1 Definition

$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$$

\]

- **Open-circuit measurements**:

\[

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}, \quad Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$$

\]

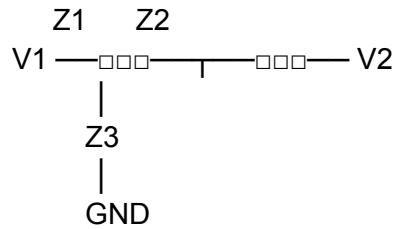
\[

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}, \quad Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

\]

7.3.2 Example: Series Impedance Network

...



...

- **Z-Matrix**:

\[

\begin{bmatrix}

$$Z_1 + Z_3 \quad Z_3$$

$$Z_3 \quad Z_2 + Z_3$$

\end{bmatrix}

\]

7.4 Admittance (Y) Parameters

7.4.1 Definition

\[

\begin{cases}

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

\end{cases}

\]

- **Short-circuit measurements**:

\[

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}, \quad Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

\]

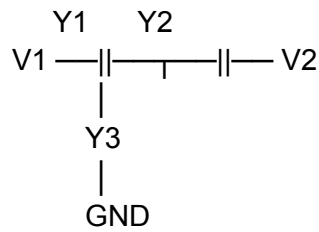
\[

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}, \quad Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

\]

7.4.2 Example: Pi Network

...



...

- **Y-Matrix**:

\[

\begin{bmatrix}

$Y_1 + Y_3$ & $-Y_3$ \\\

$-Y_3$ & $Y_2 + Y_3$

\end{bmatrix}

\]

7.5 Hybrid (h) Parameters

7.5.1 Definition

\[

\begin{cases}

$V_1 = h_{11} I_1 + h_{12} V_2$ \\\

$I_2 = h_{21} I_1 + h_{22} V_2$

\end{cases}

\]

- **Common in transistor modeling (BJT, MOSFET small-signal analysis)**

- **Measurements**:

- h_{11} = Input impedance ($V_2 = 0$)

- h_{21} = Current gain ($V_2 = 0$)

- h_{12} = Reverse voltage gain ($I_1 = 0$)

- h_{22} = Output admittance ($I_1 = 0$)

7.5.2 Example: Transistor h-Parameters

| **Parameter** | **Meaning** | **Typical BJT Value** |

|-----|-----|-----|

| h_{11} | Input impedance (h_{ie}) | $1k\Omega$ |

| h_{12} | Reverse voltage gain (h_{re}) | 10^{-4} |

| h_{21} | Current gain (h_{fe}) | 100 |

| h_{22} | Output admittance (h_{oe}) | $10\mu S$ |

7.6 Transmission (ABCD) Parameters

7.6.1 Definition

```
\[
\begin{cases}
V_1 = A V_2 - B I_2 \\
I_1 = C V_2 - D I_2
\end{cases}
\]
```

- **Used in cascaded networks (e.g., filters, transmission lines)**

- **Key Property**:

- For cascaded networks, **ABCD matrices multiply**.

7.6.2 Example: Series Impedance

...

V1 —□□□— V2
 Z

...

- **ABCD Matrix**:

```
\[
\begin{bmatrix}
1 & Z \\
0 & 1
\end{bmatrix}
\]
```

7.7 Applications of Two-Port Networks

7.7.1 Amplifier Analysis

- **Voltage Gain**:

```
\[
A_V = \frac{V_2}{V_1} \quad \text{(using Z/Y/h-parameters)}
\]
```

- **Input/Output Impedance**:

```
\[
Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L}
\]
```

7.7.2 Filter Design

- **Image Parameter Method**: Uses ABCD parameters for ladder networks.

7.7.3 RF and Microwave Circuits

- **S-Parameters**:

- Describe reflection/transmission in high-frequency systems.

7.8 Summary Table: Two-Port Parameter Conversion

Parameter	Z	Y	h	ABCD
Z	Y^{-1}	$\begin{bmatrix} \Delta_h/h_{22} & h_{12}/h_{22} \\ 1/h_{22} \end{bmatrix}$	$\begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix}$	$\begin{bmatrix} -h_{21}/h_{22} & 1/h_{22} \end{bmatrix}$
Y	Z^{-1}	$\begin{bmatrix} 1/h_{11} & -h_{12}/h_{11} \\ \Delta_h/h_{11} \end{bmatrix}$	$\begin{bmatrix} D/B & -\Delta_T/B \\ -1/B & A/B \end{bmatrix}$	$\begin{bmatrix} h_{21}/h_{11} & -h_{12}/h_{11} \end{bmatrix}$

Key Notes:

- $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$
- $\Delta_T = AD - BC$

7.9 Laboratory Experiment: Measuring Z-Parameters

7.9.1 Setup

- Circuit**: Passive L-network (R + C).
- Measurements**:
 - Open-circuit $(V_1/I_1) \rightarrow (Z_{11}, Z_{21})$
 - Open-circuit $(V_2/I_2) \rightarrow (Z_{12}, Z_{22})$

7.9.2 Expected Results

- **Reciprocal Network**: $Z_{12} = Z_{21}$.
- **Symmetrical Network**: $Z_{11} = Z_{22}$.

7.10 Summary

- Two-port networks** model input-output behavior of linear systems.
- Six parameter sets** (Z, Y, h, g, ABCD, S) describe different relationships.
- Applications**: Amplifiers, filters, transmission lines, RF circuits.