Chapter 7: Two-Port Networks - Basic Concepts and Parameters

7.1 Introduction to Two-Port Networks
- **Definition**:
  - A **two-port network** is an electrical circuit with **two pairs of terminals** (input and output ports), used to model amplifiers, filters, and transmission lines.
- **Key Assumptions**:
  - Linear time-invariant (LTI) system
  - No independent sources inside the network
  - Port currents satisfy $I_1 = I_1'$ and $I_2 = I_2'$ (current entering = current leaving)

7.2 Two-Port Network Representations
7.2.1 General Structure
```
     I1        I2
     →          →
   +-------+  +-------+
V1 ──┤       ├────┤       ├── V2
   │  N/W  │  │  N/W  │
   +-------+  +-------+
     ←          ←
     I1'       I2'
```

7.2.2 Parameter Types
Two-port networks can be described using **six parameter sets**:
1. **Impedance (Z) Parameters**
2. **Admittance (Y) Parameters**
3. **Hybrid (h) Parameters**
4. **Inverse Hybrid (g) Parameters**
5. **Transmission (ABCD) Parameters**
6. **Scattering (S) Parameters (for RF circuits)**

7.3 Impedance (Z) Parameters
7.3.1 Definition
$$
\begin{cases}
V_1 = Z_{11} I_1 + Z_{12} I_2 \\
V_2 = Z_{21} I_1 + Z_{22} I_2
\end{cases}
$$

\]
- **Open-circuit measurements**:
  \[
  Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}, \quad Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}
  \]
  \[
  Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}, \quad Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}
  \]

7.3.2 Example: Series Impedance Network
```
    Z1      Z2
V1 ──□□□──┬──□□□── V2
       |
      Z3
       |
      GND
```
- **Z-Matrix**:
  \[
  \begin{bmatrix}
  Z_1 + Z_3 & Z_3 \\
  Z_3 & Z_2 + Z_3
  \end{bmatrix}
  \]

7.4 Admittance (Y) Parameters
7.4.1 Definition
\[
\begin{cases}
I_1 = Y_{11} V_1 + Y_{12} V_2 \\
I_2 = Y_{21} V_1 + Y_{22} V_2
\end{cases}
\]
- **Short-circuit measurements**:
  \[
  Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}, \quad Y_{12} = \left. \frac{I_1}{V_2}
\right|_{V_1=0}
  \]
  \[
  Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}, \quad Y_{22} = \left. \frac{I_2}{V_2}
\right|_{V_1=0}

\]

7.4.2 Example: Pi Network
```
    Y1      Y2
V1 ──||──┬──||── V2
         |
        Y3
         |
        GND
```
- **Y-Matrix**:
  $$
  \begin{bmatrix}
  Y_1 + Y_3 & -Y_3 \\
  -Y_3 & Y_2 + Y_3
  \end{bmatrix}
  $$

7.5 Hybrid (h) Parameters
7.5.1 Definition
$$
\begin{cases}
V_1 = h_{11} I_1 + h_{12} V_2 \\
I_2 = h_{21} I_1 + h_{22} V_2
\end{cases}
$$
- **Common in transistor modeling (BJT, MOSFET small-signal analysis)**
- **Measurements**:
  - h_{11} = Input impedance ($V_2 = 0$)
  - h_{21} = Current gain ($V_2 = 0$)
  - h_{12} = Reverse voltage gain ($I_1 = 0$)
  - h_{22} = Output admittance ($I_1 = 0$)

7.5.2 Example: Transistor h-Parameters
Parameter	**Meaning**	**Typical BJT Value**
h_{11}	Input impedance (h_{ie})	1kΩ
h_{12}	Reverse voltage gain (h_{re})	10^{-4}
h_{21}	Current gain (h_{fe})	100
h_{22}	Output admittance (h_{oe})	10μS

7.6 Transmission (ABCD) Parameters
7.6.1 Definition
$$
\begin{cases}
V_1 = A V_2 - B I_2 \\
I_1 = C V_2 - D I_2
\end{cases}
$$
- **Used in cascaded networks (e.g., filters, transmission lines)**
- **Key Property**:
  - For cascaded networks, **ABCD matrices multiply**.

7.6.2 Example: Series Impedance
```
V1 ──□□□── V2
      Z
```
- **ABCD Matrix**:
  $$
  \begin{bmatrix}
  1 & Z \\
  0 & 1
  \end{bmatrix}
  $$

7.7 Applications of Two-Port Networks
7.7.1 Amplifier Analysis
- **Voltage Gain**:
  $$
  A_V = \frac{V_2}{V_1} \quad \text{(using Z/Y/h-parameters)}
  $$
- **Input/Output Impedance**:
  $$
  Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L}
  $$

7.7.2 Filter Design
- **Image Parameter Method**: Uses ABCD parameters for ladder networks.

7.7.3 RF and Microwave Circuits

- **S-Parameters**:
  - Describe reflection/transmission in high-frequency systems.

7.8 Summary Table: Two-Port Parameter Conversion

Parameter	**Z**	**Y**	**h**	**ABCD**
Z	-	Y^{-1}	$\begin{bmatrix} \Delta_h/h_{22} & h_{12}/h_{22} \\ -h_{21}/h_{22} & 1/h_{22} \end{bmatrix}$	$\begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix}$
Y	Z^{-1}	-	$\begin{bmatrix} 1/h_{11} & -h_{12}/h_{11} \\ h_{21}/h_{11} & \Delta_h/h_{11} \end{bmatrix}$	$\begin{bmatrix} D/B & -\Delta_T/B \\ -1/B & A/B \end{bmatrix}$

Key Notes:
- $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$
- $\Delta_T = AD - BC$

7.9 Laboratory Experiment: Measuring Z-Parameters
7.9.1 Setup
1. **Circuit**: Passive L-network (R + C).
2. **Measurements**:
   - Open-circuit V_1/I_1 → Z_{11}, Z_{21}
   - Open-circuit V_2/I_2 → Z_{12}, Z_{22}

7.9.2 Expected Results
- **Reciprocal Network**: $Z_{12} = Z_{21}$.
- **Symmetrical Network**: $Z_{11} = Z_{22}$.

7.10 Summary
1. **Two-port networks** model input-output behavior of linear systems.
2. **Six parameter sets** (Z, Y, h, g, ABCD, S) describe different relationships.
3. **Applications**: Amplifiers, filters, transmission lines, RF circuits.